

Particle production in the oscillating inflation model

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We investigate the particle production of a scalar field χ coupled to an inflaton field ϕ ($g^2\phi^2\chi^2/2$) in the *oscillating inflation* model, which was recently proposed by Damour and Mukhanov. Although the fluctuation of the ϕ field can be effectively enhanced during a stage of the oscillating inflation, the maximum fluctuation is suppressed as the critical value ϕ_c which indicates the scale of the core part of the inflaton potential decreases, in taking into account the back reaction effect of created particles. As for the χ particle production, we find that larger values of the coupling constant g are required to lead to an efficient parametric resonance with the decrease of ϕ_c , because an effective mass of inflaton around the minimum of its potential becomes larger. However, it is possible to generate the superheavy χ particle whose mass is greater than 10^{14} GeV, which would result in an important consequence for the GUT baryogenesis.

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I. INTRODUCTION

Inflationary cosmology is one of the most reliable concepts to describe the early stage of the universe [1]. This paradigm not only gives the solution to a number of shortcomings of the standard big bang cosmology, but also provides the density perturbation that may be responsible for the structure formation [2]. During the inflationary stage, a scalar field ϕ known as *inflaton* is slowly rolling down toward a minimum of its potential. Inflation ends when the kinetic energy of the inflaton becomes comparable to the potential energy. After that, the inflaton field begins to oscillate around the minimum of its potential and produces elementary particles. This process by which the energy of the inflaton field is transferred to other particles is called reheating. The original scenario of reheating was considered in Ref. [3] based on the perturbation theory, adding the phenomenological decay term to the equation of the inflaton field. However, since the production of a grand unified theory (GUT) scale boson is kinematically forbidden in this model, the GUT scale baryogenesis does not work well in this scenario.

It was recently recognized that the reheating process begins by an explosive particle production called *preheating* [4,5]. During this stage, the fluctuation of produced particles grows quasi-exponentially by parametric resonance. The efficiency of resonance depends on the model of the inflation. The chaotic inflation is one of the most efficient model for the development of the fluctuation. In the case of the massive inflaton potential $V(\phi) = m^2\phi^2/2$, another scalar field χ coupled to inflaton with a coupling $g^2\phi^2\chi^2/2$ can be enhanced in a certain range of the coupling constant [6–8]. With the coupling of $g \gtrsim 10^{-4}$, resonance turns on from a broad resonance regime and the fluctuation of the χ particle increases overcoming the diluting effect by the cosmic expansion. In the massless inflaton potential $\lambda\phi^4/4$, the growth of the inflaton fluctuation occurs even if we do not introduce another field χ coupled to inflaton [9–13]. In this case, however, since the resonance band is restricted to be narrow, the transfer of energy from the homogeneous inflaton to the fluctuation does not occur sufficiently. In the two-field model of $V(\phi, \chi) = \lambda\phi^4/4 + g^2\phi^2\chi^2/2$, the

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χ particle production is typically more efficient than the production of the ϕ particle [14]. As for other models of inflation, several authors considered preheating such as the hybrid inflation [15], and the higher curvature inflation [16].

Recently it has been recognized by Damour and Mukhanov [17] that inflation occurs even in the oscillating stage for the non-convex type potential $V(\phi)$ where $d^2V/d\phi^2$ is negative in the regime not too far from the core of the potential. In spite of the rapid oscillation of the inflaton field, inflation takes place during which the field moves in flat regions outside the core part. Liddle and Mazumdar [18] called it *oscillating inflation* and numerically calculated the revised number of e -foldings. The amount of inflation becomes larger with the decrease of the critical value ϕ_c which determines the scale of the convex core of the potential. However, it is difficult to induce a sufficient inflation only by the oscillating inflation even when ϕ_c is lowered to the electro-weak scale $\phi_c = 10^{-17} M_{\text{Pl}}$ [18]. Since the ordinary inflation takes place in the slow-roll regime before the oscillating inflation, the total amount of e -folding is mostly due to this preceding inflation and the contribution of the oscillating inflation is added to some extent. The energy scale of the potential can be determined in the usual manner by fitting the density perturbation observed by the Cosmic Background Explorer (COBE) satellite.

As was pointed out by Damour and Mukhanov, since the square of the effective mass of the inflaton field $m_\phi^2 \equiv d^2V/d\phi^2$ is mostly negative during the oscillating inflation, the fluctuation of inflaton can be strongly amplified. Taruya [19] considered the evolution of the fluctuation including the metric perturbation in the single field model during the oscillating inflation. Although the super-horizon modes ($k \rightarrow 0$) are not relevantly amplified, the growth of the modes inside the Hubble horizon occurs significantly. In the case of the single field, since the curvature perturbation on the comoving slice remains constant in the long wavelength limit, we can not expect the significant growth of the long-wave perturbation. It was also suggested by Damour and Mukhanov that another field χ coupled to inflaton would be resonantly amplified during the oscillating inflation and the superheavy GUT scale bosons are expected to be generated. The structure of resonance for the χ particle is different from that of the ϕ particle, and we should make clear the parameter range of the coupling constant g where the χ particle production occurs sufficiently. Since the effective mass of the inflaton around the minimum of the potential depends on the critical value ϕ_c , it is also important to investigate how the difference of the shape of the potential would affect the χ particle production. If the χ particle is efficiently created in the oscillating inflation model, this would affect a nonthermal phase transition [20], and topological defect formation [21]. Also, the possibility of the production of the superheavy particle would make the baryogenesis at the GUT scale possible [22]. In this paper, we consider the production of the χ particle as well as the ϕ particle during the oscillating inflation and the subsequent oscillating stage. We make use of the Hartree approximation in order to include the back reaction effect of created particles which is absent in Ref. [19]. Even in the case where the resonance band is very broad, the back reaction effect plays a crucial role to terminate the growth of the fluctuation. We will discuss how the maximum fluctuations of ϕ and χ particles depend on the critical value of ϕ_c and the coupling constant g .

This paper is organized as follows. In the next section, we introduce basic equations based on the Hartree approximation in the oscillating inflation model. In Sec. III, the particle production in this model is investigated by making use of the numerical results. We study how the fluctuations of the ϕ and χ fields grow during the oscillating inflation

and the subsequent oscillating stage. We give our discussions and conclusions in the final section.

II. THE BASIC EQUATIONS

We investigate a model where an inflaton field ϕ is coupled to a scalar field χ [23],

$$\mathcal{L} = \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2}(\nabla\phi)^2 - V(\phi) - \frac{1}{2}(\nabla\chi)^2 - \frac{1}{2}m_\chi^2\chi^2 - \frac{1}{2}g^2\phi^2\chi^2 \right], \quad (2.1)$$

where $\kappa^2/8\pi \equiv G = M_{\text{pl}}^{-2}$ is Newton's gravitational constant, R is a scalar curvature, m_χ is a mass of the χ field, and g is a coupling constant. We consider an inflaton potential $V(\phi)$ proposed by Damour and Mukhanov [17], which is described by

$$V(\phi) = \frac{M^4}{q} \left[\left(\frac{\phi^2}{\phi_c^2} + 1 \right)^{q/2} - 1 \right], \quad (2.2)$$

where M is a mass which is constrained by the primordial density perturbation observed by the COBE satellite, q is a dimensionless parameter greater than zero, and ϕ_c is a critical value of the inflaton field which determines the scale of the core part of the potential (See Fig. 1). Note that the potential (2.2) is a toy model which is not directly related with a real theory of physics, although supergravity and superstring models may give rise to some non-convex type potentials [17]. In the case of $\phi^2 \gg \phi_c^2$, the potential is approximately written by $V(\phi) \approx Aq^{-1}(\phi/\phi_c)^q$, and $d^2V/d\phi^2$ becomes negative for $q < 1$. Inflation takes place in the usual manner while inflaton slowly moves in this flat region. This conventional slow-roll inflation is followed by the *oscillating inflation*, which means that inflation continues to occur during the oscillating stage of inflaton while it evolves in the region of $\phi^2 \gtrsim \phi_c^2$. Since the amplitude of the ϕ field gradually decreases due to the adiabatic expansion of the universe, the ϕ field is finally trapped in the core region of the potential (i.e. $\phi^2 \lesssim \phi_c^2$) and the oscillating inflation ceases. After that, the universe enters the ordinary reheating stage, in which the potential (2.2) is described by the massive inflaton potential.

Hereafter, we study the dynamics of the system in the flat Friedmann-Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2, \quad (2.3)$$

where $a(t)$ is the scale factor, and t is the cosmic time coordinate.

Let us consider the equations of motion based on the Hartree factorization in the oscillating inflation model. We decompose the inflaton field into the homogeneous and fluctuational parts as

$$\phi(t, \mathbf{x}) = \phi_0(t) + \delta\phi(t, \mathbf{x}), \quad (2.4)$$

where the fluctuational part satisfies the tadpole condition

$$\langle \delta\phi(t, \mathbf{x}) \rangle = 0. \quad (2.5)$$

In order to study the quantum particle creation, we expand $\delta\phi$ and χ fields as

$$\delta\phi = \frac{1}{(2\pi)^{3/2}} \int \left(a_k \delta\phi_k(t) e^{-i\mathbf{k}\cdot\mathbf{x}} + a_k^\dagger \delta\phi_k^*(t) e^{i\mathbf{k}\cdot\mathbf{x}} \right) d^3\mathbf{k}, \quad (2.6)$$

$$\chi = \frac{1}{(2\pi)^{3/2}} \int \left(a_k \chi_k(t) e^{-i\mathbf{k}\cdot\mathbf{x}} + a_k^\dagger \chi_k^*(t) e^{i\mathbf{k}\cdot\mathbf{x}} \right) d^3\mathbf{k}, \quad (2.7)$$

where a_k and a_k^\dagger are the annihilation and creation operators respectively.

Then, the equations of motion for inflaton are expressed by imposing the Hartree factorization [24] as

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + \sum_{n=0}^{\infty} \frac{1}{2^n n!} \langle \delta\phi^2 \rangle^n V^{(2n+1)}(\phi_0) + g^2 \langle \chi^2 \rangle \phi_0 = 0, \quad (2.8)$$

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \left[\frac{k^2}{a^2} + \sum_{n=0}^{\infty} \frac{1}{2^n n!} \langle \delta\phi^2 \rangle^n V^{(2n+2)}(\phi_0) + g^2 \langle \chi^2 \rangle \right] \delta\phi_k = 0, \quad (2.9)$$

where $H \equiv \dot{a}/a$, $V^n(\phi_0) \equiv \delta^n V(\phi_0)/\delta\phi_0^n$, and $\langle \delta\phi^2 \rangle$, $\langle \chi^2 \rangle$ are defined by

$$\langle \delta\phi^2 \rangle = \frac{1}{2\pi^2} \int k^2 |\delta\phi_k|^2 dk, \quad (2.10)$$

$$\langle \chi^2 \rangle = \frac{1}{2\pi^2} \int k^2 |\chi_k|^2 dk. \quad (2.11)$$

Note that the $V^{(2)}(\phi_0)$ term in Eq. (2.9) is the leading term for the development of the fluctuation $\langle \delta\phi^2 \rangle$. In the present model, this term is negative in most stages of the oscillating inflation, and the growth of the fluctuation can be expected during this stage. As ϕ particles are produced, however, the back reaction effect by the growth of $\langle \delta\phi^2 \rangle$ plays an important role. The development of the fluctuation is suppressed by the ϕ particle production itself.

With regard to the χ field, this satisfies the following equation

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left[\frac{k^2}{a^2} + m_\chi^2 + g^2(\phi_0^2 + \langle \delta\phi^2 \rangle) \right] \chi_k = 0. \quad (2.12)$$

The $g^2\phi_0^2$ term leads to the parametric amplification of χ particles when the ϕ_0 field oscillates around the minimum of its potential. The strength of resonance depends on the coupling constant g and the amplitude of the ϕ_0 field. Moreover, the effective mass m_ϕ of the ϕ_0 field is also important as we will show later. Since m_ϕ is closely related to the shape around the core region, the development of the fluctuation $\langle \chi^2 \rangle$ depends on the critical value ϕ_c . In the case where the growth rate of the χ fluctuation is large, back reaction effects of χ particles as well as ϕ particles finally shut off the parametric resonance.

Next, the evolution of the scale factor is described by

$$\begin{aligned} \left(\frac{\dot{a}}{a} \right)^2 &= \frac{\kappa^2}{3} \left[\frac{1}{2} \dot{\phi}_0^2 + \frac{1}{2} \langle \delta\dot{\phi}^2 \rangle + \frac{1}{2a^2} \langle (\nabla\phi)^2 \rangle + \sum_{n=0}^{\infty} \frac{1}{2^n n!} \langle \delta\phi^2 \rangle^n V^{2n}(\phi_0) \right. \\ &\quad \left. + \frac{1}{2} \langle \dot{\chi}^2 \rangle + \frac{1}{2a^2} \langle (\nabla\chi)^2 \rangle + \frac{1}{2} \{ m_\chi^2 + g^2(\phi_0^2 + \langle \delta\phi^2 \rangle) \} \langle \chi^2 \rangle \right], \end{aligned} \quad (2.13)$$

where

$$\langle \delta\dot{\phi}^2 \rangle = \frac{1}{2\pi^2} \int k^2 |\delta\dot{\phi}_k|^2 dk, \quad (2.14)$$

$$\langle (\nabla\phi)^2 \rangle = \frac{1}{2\pi^2} \int k^4 |\delta\phi_k|^2 dk, \quad (2.15)$$

$$\langle (\nabla\chi)^2 \rangle = \frac{1}{2\pi^2} \int k^4 |\chi_k|^2 dk. \quad (2.16)$$

Before analyzing the above equation of motion, we first discuss the evolution of the scale factor and the homogeneous inflaton field neglecting the fluctuational terms. Then, Eqs. (2.8) and (2.13) are approximately written as

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + V_{,\phi_0}(\phi_0) \approx 0, \quad (2.17)$$

$$\left(\frac{\dot{a}}{a}\right)^2 \approx \frac{\kappa^2}{3} \left[\frac{1}{2} \dot{\phi}_0^2 + V(\phi_0) \right]. \quad (2.18)$$

We can understand the mean behavior of the scale factor and the inflaton field by ignoring the core region of the potential. For the case of $\phi_0^2 \gg \phi_c^2$, making use of the time averaged relation $\langle \dot{\phi}^2 \rangle_T = q \langle V(\phi) \rangle_T$ in Eqs. (2.17) and (2.18), we find the following approximate relation [17]

$$a \propto t^{(q+2)/3q}, \quad (2.19)$$

$$\tilde{\phi}_0 \propto t^{-2/q}, \quad (2.20)$$

where $\tilde{\phi}_0$ is the amplitude of the ϕ_0 field. Note that the universe expands acceleratedly for $0 < q < 1$. Hereafter, we consider the case of $0 < q < 1$. Since inflation takes place for $\phi_0^2 \gtrsim \phi_c^2$, the amount of inflation becomes larger with the decrease of ϕ_c . Damour and Mukhanov estimated the number of e -foldings as

$$N \equiv \ln \frac{a_f}{a_s} \approx \frac{2+q}{6} \ln \frac{\phi_0(t_s)}{\phi_c}, \quad (2.21)$$

where the subscripts s and f denote the end of the slow-roll and the end of the oscillating inflation respectively. The slow-roll inflation ends when the slow-roll parameter $\epsilon \equiv (V_{,\phi_0}/V)^2/2\kappa^2$ becomes of order unity. For the case of $\phi_0^2 \gg \phi_c^2$, since ϵ can be written as

$$\epsilon \approx \frac{q^2 M_{\text{pl}}^2}{16\pi \phi_0^2}, \quad (2.22)$$

$\phi_0(t_s)$ is estimated by setting $\epsilon = 1$ as

$$\phi_0(t_s) \approx \frac{q}{\sqrt{16\pi}} M_{\text{pl}}. \quad (2.23)$$

We find from Eqs. (2.21) and (2.23) that the number of e -foldings during the oscillating inflation is small compared with the total needed amount of inflation $N_t \gtrsim 60$. For example, when $\phi_c = 10^{-6} M_{\text{pl}}$ and $q = 0.1$, $N = 3.3$. Even if ϕ_c is the electro-weak scale $\phi_c = 10^{-17} M_{\text{pl}}$, $N = 12.2$ for $q = 0.1$. As was confirmed by numerical calculations in Ref. [18], the number of e -foldings becomes smaller as q approaches zero or unity for the fixed value of ϕ_c . Hence even if we change the values of q in the range of $0 < q < 1$, the contribution to the amount of inflation due to the oscillating inflation is still small. This means that the preceding inflation by the ordinary slow-roll is expected to contribute to most of the number of e -foldings needed to solve cosmological puzzles.

Let us estimate the energy scale of the potential (2.2) which is constrained by the density perturbation observed by the COBE satellite. First, we consider the value of $\phi_0 (= \phi_0(t_i))$ at the epoch of horizon exit when physical scales

crossed outside the Hubble radius 50 e -foldings before the start of the oscillating inflation. Calculating the number of e -foldings

$$N = -\kappa^2 \int_{\phi_0(t_i)}^{\phi_0(t_s)} \frac{V}{V_{,\phi_0}} d\phi_0, \quad (2.24)$$

in the present model with the condition of $\phi_0^2 \gg \phi_c^2$, we obtain

$$N \approx \frac{4\pi}{qM_{\text{pl}}^2} [\phi_0^2(t_i) - \phi_0^2(t_s)]. \quad (2.25)$$

Then the value of $\phi_0(t_i)$ is approximately estimated as

$$\phi_0(t_i) \approx \sqrt{\frac{qN}{4\pi}} M_{\text{pl}}, \quad (2.26)$$

where we neglected the contribution of the $\phi_0(t_s)$ term. The square of the amplitude of the density perturbation can be calculated for $\phi_0^2 \gg \phi_c^2$ as [25]

$$\delta_H^2 = \frac{32}{75} \frac{V(\phi_0(t_i))}{M_{\text{pl}}^4} \epsilon^{-1}(\phi_0(t_i)) \approx \frac{512\pi}{75} \frac{M^4}{q^3 M_{\text{pl}}^6} \frac{\phi_0^{q+2}(t_i)}{\phi_c^q}. \quad (2.27)$$

Then the mass M is constrained to be

$$M \approx \left[\frac{75}{128} \frac{q^2}{N} \left(\frac{\phi_c}{M_{\text{pl}}} \sqrt{\frac{4\pi}{qN}} \right)^q \delta_H^2 \right]^{1/4} M_{\text{pl}}. \quad (2.28)$$

The COBE data requires $\delta_H \approx 2 \times 10^{-5}$. Setting $N = 50$, we can estimate the mass M as two functions of q and ϕ_c . The mass M is weakly dependent on the value of ϕ_c for the fixed value of q . For example, for $\phi_c = 10^{-4} M_{\text{pl}}$ and $q = 0.1$, $M = 3.74 \times 10^{-4} M_{\text{pl}}$; and for $\phi_c = 10^{-6} M_{\text{pl}}$ and $q = 0.1$, $M = 3.33 \times 10^{-4} M_{\text{pl}}$. In the next section, we use the value of M which is obtained by Eq. (2.28), and consider the growth of the fluctuation during and after the oscillating inflation.

III. PARTICLE PRODUCTION IN THE OSCILLATING INFLATION MODEL

In this section, we investigate the particle production of ϕ and χ fields with the potential (2.2). Defining new scalar fields $\varphi_0 \equiv a^{3/2} \phi_0$, $\delta\varphi_k \equiv a^{3/2} \delta\phi_k$, and $X_k \equiv a^{3/2} \chi_k$, Eqs. (2.8), (2.9), and (2.12) can be rewritten as

$$\ddot{\varphi}_0 + \omega_{\varphi_0}^2 \varphi_0 = 0, \quad (3.1)$$

$$\delta\ddot{\varphi}_k + \omega_{\delta\varphi_k}^2 \delta\varphi_k = 0, \quad (3.2)$$

$$\ddot{X}_k + \omega_{X_k}^2 X_k = 0, \quad (3.3)$$

with

$$\begin{aligned} \omega_{\varphi_0}^2 &\equiv \frac{M^4}{\phi_c^2} \left(\frac{\phi_0^2}{\phi_c^2} + 1 \right)^{\frac{1}{2}q-1} + \frac{1}{2} \frac{M^4}{\phi_c^4} \langle \delta\phi^2 \rangle \left(\frac{\phi_0^2}{\phi_c^2} + 1 \right)^{\frac{1}{2}q-3} (q-2) \left\{ (q-1) \frac{\phi_0^2}{\phi_c^2} + 3 \right\} \\ &+ g^2 \langle \chi^2 \rangle - \frac{3}{4} \left(\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right), \end{aligned} \quad (3.4)$$

$$\begin{aligned}\omega_{\delta\varphi_k}^2 &\equiv \frac{k^2}{a^2} + \frac{M^4}{\phi_c^2} \left(\frac{\phi_0^2}{\phi_c^2} + 1 \right)^{\frac{1}{2}q-2} \left\{ (q-1) \frac{\phi_0^2}{\phi_c^2} + 1 \right\} + \frac{1}{2} \langle \delta\phi^2 \rangle \frac{M^4}{\phi_c^4} \left(\frac{\phi_0^2}{\phi_c^2} + 1 \right)^{\frac{1}{2}q-4} \\ &\times (q-2) \left\{ (q-1)(q-3) \frac{\phi_0^4}{\phi_c^4} + 6(q-3) \frac{\phi_0^2}{\phi_c^2} + 3 \right\} + g^2 \langle \chi^2 \rangle - \frac{3}{4} \left(\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right),\end{aligned}\quad (3.5)$$

$$\omega_{X_k}^2 \equiv \frac{k^2}{a^2} + m_\chi^2 + g^2(\phi_0^2 + \langle \delta\phi^2 \rangle) - \frac{3}{4} \left(\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right), \quad (3.6)$$

where we have considered the $\langle \delta\phi^2 \rangle^n$ terms up to $n = 1$. The higher order terms do not significantly contribute to the evolution of the system. We find from Eq. (3.4) that the frequency of the φ_0 field changes even during one oscillation of the φ_0 field. At the stage when the particles are not sufficiently produced ($\langle \delta\phi^2 \rangle, \langle \chi^2 \rangle \ll \tilde{\phi}_0^2$), the second and third terms in Eq. (3.4) are negligible. Moreover, since the last term in Eq. (3.4) is estimated by using Eqs. (2.17) and (2.18) as

$$-\frac{3}{4} \left(\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) \approx \frac{3}{4} \kappa^2 \left[\frac{1}{2} \dot{\phi}_0^2 - V(\phi_0) \right], \quad (3.7)$$

this term is also negligible compared with the first term in Eq. (3.4) for the typical values of ϕ_c ($\lesssim 10^{-3} M_{\text{Pl}}$). Then the effective mass of the inflaton in the first stage of the oscillating inflation for two limiting cases $\phi_0^2 \gg \phi_c^2$ and $\phi_0^2 \ll \phi_c^2$ is given by

$$m_\phi = \begin{cases} \frac{M^2}{\phi_c} \left(\frac{\phi_c}{\phi_0} \right)^{1-\frac{q}{2}} & (\phi_0^2 \gg \phi_c^2) \\ \frac{M^2}{\phi_c} & (\phi_0^2 \ll \phi_c^2) \end{cases} \quad (3.8)$$

Note that m_ϕ for $\phi_0^2 \gg \phi_c^2$ is smaller than m_ϕ for $\phi_0^2 \ll \phi_c^2$ because the potential is flat for $\phi_0^2 \gg \phi_c^2$. As time passes and particles are produced, back reaction effects due to the increase of $\langle \delta\phi^2 \rangle$ and $\langle \chi^2 \rangle$ become relevant and the coherent oscillation of the φ_0 field is prevented by the change of the effective mass m_ϕ .

In the frequency (3.5) of the $\delta\varphi_k$ field, since we consider the situation of $0 < q < 1$, the second term becomes negative in the region of $\phi_0^2 \gtrsim \phi_c^2$. Moreover, in the small vicinity of $\phi_0^2 \lesssim \phi_c^2$, this term rapidly grows and takes large positive value $\sim M^4/\phi_c^2$. These peculiar behaviors of the frequency lead to the enhancement of the ϕ particle during the stage of the oscillating inflation.

As for the χ field, since the oscillation of the ϕ_0 field is different from that of the ordinary inflaton potential, the increase of $\langle \chi^2 \rangle$ occurs in a different way. In this case, although analytic investigations of the evolution of the χ particle are rather difficult, we shall examine the parameter range of the critical value ϕ_c and the coupling constant g where the χ particle production takes place efficiently.

With regard to the initial conditions of the fluctuation, we choose the states which correspond to the conformal vacuum as

$$\delta\varphi_k(0) = \frac{1}{\sqrt{2\omega_{\delta\varphi_k(0)}}}, \quad \delta\dot{\varphi}_k(0) = -i\omega_{\delta\varphi_k(0)}\delta\varphi_k(0), \quad (3.9)$$

$$X_k(0) = \frac{1}{\sqrt{2\omega_{X_k(0)}}}, \quad \dot{X}_k(0) = -i\omega_{X_k(0)}X_k(0), \quad (3.10)$$

We investigate the evolution of the fluctuations of ϕ and χ fields with those initial conditions as the semiclassical problem. In order to clarify the situation we consider, we first investigate the case when the coupling g is zero (i.e. one-field case) and next investigate the case of $g \neq 0$ (two-field case).

A. The case of $g = 0$

In this subsection, we study the evolution of the inflaton quanta in the case of $g = 0$. This case was originally considered by Taruya [19] including the metric perturbation. However, since his results are obtained neglecting the back reaction effect of created particles, we investigate how this effect would modify the growth of the fluctuation. The first thing we should notice is that the strength of the amplification of the fluctuation $\langle \delta\phi^2 \rangle$ strongly depends on the critical value ϕ_c . With the decrease of ϕ_c , since the duration of the oscillating inflation becomes longer, parametric amplification of the ϕ particle occurs more efficiently before the back reaction effect becomes significant.

We investigate two concrete cases of $\phi_c = 10^{-3}M_{\text{pl}}$ and $\phi_c = 10^{-4}M_{\text{pl}}$ with the value of $q = 0.1$ where the growth of the fluctuation can be expected. When $q = 0.1$, the initial value of ϕ_0 is estimated as $\phi_0 = 1.4 \times 10^{-2}M_{\text{pl}}$ by Eq. (2.23).

Let us first consider the evolution of the ϕ_0 field. In the case of $\phi_c = 10^{-3}M_{\text{pl}}$, since the initial value of ϕ_0 is larger than ϕ_c only by one order of magnitude, the number of e -foldings during the oscillating inflation is small [$N = 0.93$ by Eq. (2.21)]. In Fig. 2, we depict the evolution of the ϕ_0 field. The amplitude $\tilde{\phi}_0$ gradually decreases due to the expansion of the universe, and the oscillating inflation ends when $\tilde{\phi}_0$ is lowered to ϕ_c . Numerical calculations indicate that this occurs at $\bar{t} \equiv M^2 t / M_{\text{pl}} \approx 0.37$. After this, the ϕ_0 field oscillates around the core region $\phi_0^2 \lesssim \phi_c^2$, whose bare mass is given by $m_\phi = M^2/\phi_c$. In the case of $\phi_c = 10^{-4}M_{\text{pl}}$, since the oscillating inflation continues until $\tilde{\phi}_0 \approx 10^{-4}M_{\text{pl}}$, we obtain the larger value of e -foldings $N = 1.73$ compared with the case of $\phi_c = 10^{-3}M_{\text{pl}}$. Numerically, the oscillating inflation ceases at $\bar{t} \approx 0.45$ in this case.

With regard to the fluctuation of inflaton, the resonance band is very broad and the growth rate of the fluctuation becomes of order unity. We can expect the enormous amplification of the wave modes especially within the Hubble radius. We show in Fig. 3 the evolution of the $\delta\phi_k$ field for two cases of $\bar{k} \equiv k/(M^2/M_{\text{pl}}) = 0.1$ and $\bar{k} = 100$. We find that long-wave modes are not significantly enhanced compared with the modes inside the horizon scale. This result coincides with numerical calculations performed in Ref. [19] which include the metric perturbation. As was presented in Ref. [19], the effect of the metric perturbation appears in the term $2\kappa^2(V/H)$ of the equation of the Mukhanov variable $Q \equiv \delta\phi - \mathcal{R}\dot{\phi}/H$, where \mathcal{R} is the spatial curvature [See Eq. (8) in Ref. [19]]. However, since this term decreases faster than the term $V^{(2)}(\phi_0)$ in Eq. (2.9), we expect that adding this term to the equation of the fluctuation will not alter the evolution of the system. We have numerically checked that the growth of the fluctuation in Fig. 3 is almost the same as in the case where the $2\kappa^2(V/H)$ term is included. In the single-field case, there exists an exact solution for the Mukhanov variable in the long wavelength limit [See Eq. (9) in Ref. [19]], and we can confirm that the amplitude of the fluctuation remains nearly constant. On the other hand, since the instability band is broad in the present model even for the large k , the momentum modes up to $k^2/a^2 \lesssim M^4/\phi_c^2$ inside the Hubble horizon are effectively enhanced. In Fig. 3, we find that the fluctuation $\delta\phi_k$ rapidly grows at the stage of the oscillating inflation for the case of $\bar{k} = 100$. However, this increase is suppressed at $\bar{t} \approx 0.12$, before the oscillating inflation terminates at $\bar{t} \approx 0.37$. This is due to the fact that the back reaction effect of created particles becomes significant. We have numerically confirmed that the fluctuation continues to grow during the oscillating inflation if we neglect the back reaction effect as in Ref. [19]. The increase of $\langle \delta\phi^2 \rangle$ effectively changes both frequencies of φ_0 and $\delta\varphi_k$ fields as is

found by Eqs. (3.4) and (3.5), and the growth of the fluctuation finally stops when the $\langle\delta\phi^2\rangle$ terms become comparable to the preceding terms in Eqs. (3.4) and (3.5). It is expected that the back reaction effect becomes significant when $\langle\delta\phi^2\rangle$ increases of order ϕ_c^2 . In the case of $\phi_c = 10^{-3}M_{\text{pl}}$, numerical calculations show that the maximum value of the fluctuation is $\langle\delta\phi^2\rangle_f \approx 10^{-7}M_{\text{pl}}^2$ at $\bar{t} \approx 0.12$ (See Fig. 4). In this case, parametric amplification of the ϕ particle terminates when $\langle\delta\phi^2\rangle$ increases up to $\langle\delta\phi^2\rangle_f \approx 0.1\phi_c^2$. As is seen in Fig. 2, the coherent oscillation of the ϕ_0 field is hardly broken after $\bar{t} \approx 0.12$ where the variance of the ϕ particle reaches the maximum value. This means that the growth of $\langle\delta\phi^2\rangle$ affects the evolution of the $\delta\phi_k$ field more strongly than that of the ϕ_0 field.

With the decrease of ϕ_c , since the oscillating inflation continues longer, the growth rate of the fluctuation in the first stage becomes larger. However, the final variance $\langle\delta\phi^2\rangle_f$ is suppressed as ϕ_c becomes smaller. We depict in Fig. 5 the evolution of the fluctuation $\langle\delta\phi^2\rangle$ in the case of $\phi_c = 10^{-4}M_{\text{pl}}$. In this case, although the oscillating inflation continues until $\bar{t} \approx 0.4$, the growth of the fluctuation stops much earlier: $\bar{t} \approx 0.05$. Although the initial growth rate is larger than in the case of $\phi_c = 10^{-3}M_{\text{pl}}$, the production of the ϕ particle itself terminates the growth of the fluctuation. This tendency is stronger with the decrease of ϕ_c , and the maximum fluctuation becomes smaller. We have found the following relation for the typical value of $\phi_c \lesssim 10^{-3}M_{\text{pl}}$ as

$$\langle\delta\phi^2\rangle_f \approx 0.1\phi_c^2. \quad (3.11)$$

Although the ϕ particle production is possible at the initial stage of the oscillating inflation, the final variance is strongly suppressed with the decrease of ϕ_c as Eq. (3.11). After the amplitude of the inflaton field drops under $\tilde{\phi}_0 \lesssim \phi_c$, the system enters the ordinary reheating stage where the universe expands deceleratedly. In this stage, the growth of the fluctuation $\langle\delta\phi^2\rangle$ can be no longer expected because the inflaton field behaves as the massive inflaton whose mass is $m_\phi = M^2/\phi_c^2$.

In the next subsection, we consider another field χ coupled to inflaton and analyze how χ particles are produced during the oscillating inflation and subsequent oscillating phase by parametric resonance.

B. The case of $g \neq 0$

Next, we consider the production of the χ particle coupled to the inflaton field. In the ordinary picture of the slow-roll inflation, the χ particle production is inefficient during the inflationary phase. In the present model, however, since the inflaton field oscillates rapidly during the oscillating inflation, χ particles can be produced in this stage by parametric resonance.

The strength of resonance depends on the coupling g , the amplitude of ϕ_0 ($= \tilde{\phi}_0$), and the mass of the inflaton field ($= m_\phi$). In the model of the scalar field χ coupled to the massive inflaton ϕ : $V(\phi, \chi) = m^2\phi^2/2 + g^2\phi^2\chi^2/2$, the equation of the χ field is reduced to the Mathieu equation [26] at the linear stage. In this model, the resonance parameter $q \equiv g^2\tilde{\phi}^2/(4m^2)$ is an important factor in determining whether the χ particle production is efficient or not [8]. When q is sufficiently large initially as $q_i \gg 1$, parametric resonance turns on from broad resonance regimes. Although q decreases with the decrease of $\tilde{\phi}_0$ due to the cosmic expansion, the fluctuation $\langle\chi^2\rangle$ grows quasiexponentially until q drops under unity or the back reaction effect of created particles becomes significant.

In the oscillating inflation model, since the mass $m_\phi^2 = \frac{M^4}{\phi_c^2} \left(\frac{\phi_0^2}{\phi_c^2} + 1 \right)^{q/2-1}$ continually changes during one oscillation of inflaton, the φ_0 field does not oscillate sinusoidally and the method based on the Mathieu equation is not valid except when $\tilde{\phi}_0$ drops under ϕ_c . In the frequency (3.6) of the X_k field, since the last term is negligible when the χ particle production occurs sufficiently, the equation of the χ field for the case of $\langle \delta\phi^2 \rangle \ll \tilde{\phi}_0^2$ is approximately written by

$$\frac{d^2}{dz^2} X_k + \left[\frac{\bar{k}^2}{a^2} + \bar{m}_\chi^2 + \frac{g^2 \phi_0^2}{m_\phi^2} \right] X_k \approx 0, \quad (3.12)$$

where $z \equiv m_\phi t$, $\bar{k} \equiv k/m_\phi$, and $\bar{m}_\chi \equiv m_\chi/m_\phi$. When the initial value of $g^2 \phi_0^2/m_\phi^2$ is large, parametric resonance works out more efficiently. The initial value of ϕ_0 estimated by Eq. (2.23) for the typical value of q is smaller than the initial value $\phi_0 \sim 0.2\text{--}0.3M_{\text{Pl}}$ in the massive inflaton model. Moreover, as is found by Eq. (2.20), the amplitude of the ϕ_0 field decays faster than in the model of the massive inflaton with $\tilde{\phi}_0 \propto t^{-1}$.

The effective mass of the ϕ_0 field is approximately written in two asymptotic regions as Eq. (3.8), and depends on the critical value ϕ_c . The mass $m_\phi \approx M^2/\phi_c^2$ around $\phi_0^2 \lesssim \phi_c^2$ plays the more important role than that of $\phi_0^2 \gtrsim \phi_c^2$ for the development of the fluctuation, because χ particles are mainly produced in the vicinity of $\phi_0 = 0$ where the ϕ_0 field changes nonadiabatically. During one oscillation of the ϕ_0 field, m_ϕ gradually gets larger as ϕ_0 approaches the minimum of the potential for the fixed value of ϕ_c . Moreover, for the typical value of $\phi_c \lesssim 10^{-3}M_{\text{Pl}}$ where the oscillating inflation occurs, $m_\phi = M^2/\phi_c$ is greater than the mass $\sim 10^{-6}M_{\text{Pl}}$ in the model of the massive inflaton. For example, in the case of $\phi_c = 10^{-3}M_{\text{Pl}}$ and $q = 0.1$, $M^2/\phi_c = 1.57 \times 10^{-4}M_{\text{Pl}}$; for $\phi_c = 10^{-4}M_{\text{Pl}}$ and $q = 0.1$, $M^2/\phi_c = 1.40 \times 10^{-3}M_{\text{Pl}}$.

In addition to the fact that $\tilde{\phi}_0$ decreases faster than in the model of the massive inflaton, the larger mass m_ϕ results in the restriction of the coupling constant g for an efficient particle production. Since the dependence of M^2 for the value ϕ_c is weak, the mass M^2/ϕ_c monotonically increases with the decrease of ϕ_c . This means that parametric resonance does not take place unless we choose a large coupling constant g as ϕ_c decreases. When ϕ_c is fairly large as $\phi_c \gtrsim 0.1M_{\text{Pl}}$, M^2/ϕ_c becomes comparable to the mass of the massive inflaton model, but in this case the creation of ϕ particles can not be expected because of the absence of the oscillating inflationary phase. What we are interested in is the case of $\phi_c \lesssim 10^{-3}M_{\text{Pl}}$ where the oscillating inflation occurs and both of ϕ and χ particles are generated. In what follows, we examine the χ particle production in two cases of $\phi_c = 10^{-3}M_{\text{Pl}}$ and $\phi_c = 10^{-4}M_{\text{Pl}}$ for the massless χ particle, and add the discussion of the case where the mass of the χ particle is included at the final of this section.

When $\phi_c = 10^{-3}M_{\text{Pl}}$, numerical calculations indicate that the increase of $\langle \chi^2 \rangle$ can take place for $g \gtrsim 5 \times 10^{-3}$. However, parametric resonance is weak for the case of $g \lesssim 0.03$. As compared with the model of $V(\phi, \chi) = m^2 \phi^2/2 + g^2 \phi^2 \chi^2/2$ in which the χ particle production takes place for $g \gtrsim 10^{-4}$ [8], larger values of g are required even in the case of $\phi_c = 10^{-3}M_{\text{Pl}}$ where the oscillating inflation marginally occurs. Since the fluctuation $\langle \chi^2 \rangle$ does not grow well for $g \lesssim 0.03$, the evolution of the ϕ_0 field is hardly affected by the χ particle production. For the case of $g \gtrsim 0.05$, the back reaction effect of the created χ particle begins to be significant. We depict in Fig. 6 the evolution of $\langle \chi^2 \rangle$ for two cases of $g = 0.03$ and $g = 0.07$. Although parametric resonance evidently occurs for $g = 0.03$, this is rather inefficient and $\langle \chi^2 \rangle$ takes the maximum value $\langle \chi^2 \rangle_f = 2.3 \times 10^{-11}M_{\text{Pl}}^2$ at $\bar{t} = 0.177$. In this case, the χ field deviates from the resonance bands with the decrease of $\tilde{\phi}_0$ due to the cosmic expansion before the oscillating inflation ceases

at $\bar{t} \approx 0.37$. Since the maximum fluctuation of $\langle \delta\phi^2 \rangle$ is almost the same as the $g = 0$ case, the back reaction effect of ϕ particles onto the ϕ_0 field can be marginally negligible. Hence in the case of $g \lesssim 0.03$, the coherent oscillation of the ϕ_0 field is hardly prevented by the back reaction effects of both ϕ and χ particles. When $g = 0.07$, the evolution of the system shows different characteristics compared with the case of $g \lesssim 0.03$. In Fig. 7, the evolution of the ϕ_0 field for the $g = 0.07$ case is depicted. We find that the coherent oscillation of the ϕ_0 field is broken for $\bar{t} \gtrsim 0.12$, which is different from the $g = 0$ case in Fig. 2. As for the ϕ particle, the maximum fluctuation is $\langle \delta\phi^2 \rangle_f = 2.7 \times 10^{-6} M_{\text{pl}}^2$ at $\bar{t} = 0.12$. This maximum variance is greater than in the case of $g = 0$ by one order of magnitude (See Fig. 8). As was noticed in the previous subsection, the growth of $\langle \delta\phi^2 \rangle$ stops when the contribution of the third term in the frequency (3.5) becomes comparable to the second term. Since the third term mostly takes negative values, the increase of the $g^2 \langle \chi^2 \rangle$ term due to the χ particle production generally assists the ϕ particle production to continue longer. This results in the larger value of the final fluctuation $\langle \delta\phi^2 \rangle$, and the back reaction effect due to the ϕ particle production becomes relevant at $\bar{t} \approx 0.12$. This behavior is found in Fig. 7. However, since the ϕ_0 field still oscillates after the back reaction effect of the ϕ particle becomes important, the χ particle continues to be enhanced after $\bar{t} \approx 0.12$. The fluctuation of $\langle \chi^2 \rangle$ does not increase significantly in the initial stage of the oscillating inflation, because the expansion rate of the universe is large. $\langle \chi^2 \rangle$ begins to increase rapidly after $\bar{t} \approx 0.15$, and reaches the maximum value $\langle \chi^2 \rangle_f = 2.4 \times 10^{-7} M_{\text{pl}}^2$ at $\bar{t} = 0.28$. The back reaction effect of the χ particles as well as the decrease of $\tilde{\phi}_0$ terminates the parametric amplification of χ particles. When $g = 0.1$, the increase of $\langle \chi^2 \rangle$ is more significant and the final variance is $\langle \chi^2 \rangle_f = 3.3 \times 10^{-6} M_{\text{pl}}^2$. In this case, since the maximum variance is larger than in the case of $g = 0.07$ by one order of magnitude, the coherent oscillation of the ϕ_0 field is more strongly prevented by the growth of $\langle \chi^2 \rangle$. When $g \gtrsim 0.3$, the final variance is suppressed because the back reaction effect becomes quite significant. We find that $\langle \chi^2 \rangle_f$ takes the maximum value $\langle \chi^2 \rangle_{\text{max}} \approx 10^{-5} M_{\text{pl}}^2$ for $g \approx 0.3$ (See Fig. 9).

As for the momentum modes of the produced χ particle, we show the evolution of the χ_k field in two cases of $\bar{k} = 0$ and $\bar{k} = 100$ for $g = 0.1$ in Fig. 10. Since the larger k makes the χ_k field deviate from the resonance band in Eq. (3.12), higher momentum modes of the χ particle are not sufficiently produced. The low momentum modes mainly contribute to the growth of the fluctuation $\langle \chi^2 \rangle$. This property is the same as in the model of $V(\phi, \chi) = m^2 \phi^2/2 + g^2 \phi^2 \chi^2/2$ [8]. As was found in Ref. [30], there is a possibility that long wavelength modes of the metric perturbation are amplified considering the perturbed metric. This issue will be discussed elsewhere [27].

In the case of $\phi_c = 10^{-4} M_{\text{pl}}$, we need further large values of g to yield an efficient resonance. Numerically, we find that the coupling is required $g \gtrsim 0.01$ for the development of the χ fluctuation. Even when $g = 0.1$, parametric resonance is not so efficient compared with the case of $\phi_c = 10^{-3} M_{\text{pl}}$. We depict in Fig. 11 the evolution of $\langle \chi^2 \rangle$ for $g = 0.07, 0.1, 0.5$ cases respectively. When $g = 0.07$, the maximum value of the fluctuation is $\langle \chi^2 \rangle_f = 1.2 \times 10^{-10} M_{\text{pl}}^2$, which is much smaller than in the case of $\phi_c = 10^{-3} M_{\text{pl}}$ with the same value of g . As the coupling increases further, we obtain larger values of the maximum fluctuation $\langle \chi^2 \rangle_f$. However, for $g \gtrsim 0.5$, the back reaction effect of created χ particles becomes significant and the final variance is suppressed. In the case of $\phi_c = 10^{-4} M_{\text{pl}}$ and $g = 0$, the fluctuation of the ϕ field rapidly grows in the initial stage and reaches the maximum value $\langle \delta\phi^2 \rangle_f \approx 10^{-9} M_{\text{pl}}^2$ at $\bar{t} = 0.05$. Even if we take into account the interaction with the χ field, this maximum variance is hardly altered because the increase of $\langle \chi^2 \rangle$ is weak for $\bar{t} \lesssim 0.05$ as is found in Fig. 11. The back reaction effect of ϕ particles onto

the ϕ_0 field is marginally negligible as the case of $g = 0$. When $g = 0.1$, the final variance is $\langle \chi^2 \rangle_f = 2.5 \times 10^{-9} M_{\text{pl}}^2$, and the growth of the fluctuation hardly affects the evolution of the ϕ_0 field. However, in the case of $g = 0.5$, $\langle \chi^2 \rangle_f$ increases up to $\langle \chi^2 \rangle_f = 1.3 \times 10^{-6} M_{\text{pl}}^2$ at $\bar{t} \approx 0.17$. In Fig. 12, we find that the increase of χ particles prevents the coherent oscillation of the ϕ_0 field for $\bar{t} \gtrsim 0.17$. When $g \gtrsim 0.5$, since the χ particle production stops by the back reaction effect, the final variance gradually decreases with the increase of g . As a result, the maximum variance for the case of $\phi_c = 10^{-4} M_{\text{pl}}$ is $\langle \chi^2 \rangle_{\text{max}} \approx 10^{-6} M_{\text{pl}}^2$ for $g \approx 0.5$.

With the decrease of ϕ_c ($\lesssim 10^{-5} M_{\text{pl}}$), the development of the χ fluctuation is not expected unless g is unnaturally large. Moreover, in this case, although the ϕ particle production is possible in the initial stage of the oscillating inflation, the final fluctuation $\langle \delta\phi^2 \rangle_f$ is strongly suppressed. This means that it is difficult to obtain the sufficient amount of ϕ and χ particles with the natural coupling $g \lesssim 1$ in the case of $\phi_c \lesssim 10^{-5} M_{\text{pl}}$. For large critical values $\phi_c > 10^{-3} M_{\text{pl}}$, the χ particle production occurs effectively for the natural coupling. With the increase of ϕ_c , the structure of resonance for the χ field approaches that of the model $V(\phi, \chi) = m^2 \phi^2/2 + g^2 \phi^2 \chi^2/2$, and the investigation based on the Mathieu equation performed in Ref. [8] can be applied. Since the mass m_ϕ around the minimum of the inflaton potential becomes smaller, the χ particle production occurs efficiently. In this case, however, since the duration of the oscillating is too short, the ϕ particle production is hardly expected. In the case of the massless χ particle, we conclude that both ϕ and χ particles are most efficiently created in the parameter range $\phi_c = 10^{-3} - 10^{-4} M_{\text{pl}}$ for the natural coupling g .

Finally, we discuss the case where the mass of the χ particle is included. We expect that the large mass makes the χ field deviate from the resonance band by the relation (3.12). Let us consider the case of $\phi_c = 10^{-3} M_{\text{pl}}$ and $q = 0.1$. In the massless case, the χ particle production occurs relevantly for $g \gtrsim 0.05$. If the mass of the χ particle is taken into account, numerical calculations show that heavy particles up to the order of $m_\chi = 10^{14}$ GeV can be produced enough for $g \gtrsim 0.05$. In Fig. 13, the evolution of the fluctuation $\langle \chi^2 \rangle$ is depicted in two cases of $m_\chi = 3 \times 10^{14}$ GeV and $m_\chi = 1 \times 10^{15}$ GeV for $g = 0.05$. The final variance $\langle \chi^2 \rangle_f \approx 10^{-8} M_{\text{pl}}^2$ with mass $m_\chi = 3 \times 10^{14}$ GeV is almost the same as in the case of the massless χ particle with the same coupling g . This means that the massive χ particle whose mass is $m_\chi \lesssim 3 \times 10^{14}$ GeV is produced with almost the same amount as the massless χ particle for $g = 0.05$. However, the achieved amount of the produced χ particle decreases with the increase of m_χ for $m_\chi \gtrsim 3 \times 10^{14}$ GeV. In the case of $g = 0.05$, although the χ particle with mass $m_\chi \lesssim 7 \times 10^{14}$ GeV can be created, it is difficult to generate the massive particle whose mass is $m_\chi \gtrsim 1 \times 10^{15}$ GeV (See Fig. 13). In the model of $V(\phi, \chi) = m^2 \phi^2/2 + g^2 \phi^2 \chi^2/2$, analytic investigations show that the χ particle which satisfies the condition $m_\chi \lesssim \sqrt{g} 10^{15}$ GeV can be generated [5, 22]. When $m_\chi = 10^{14}$ GeV, numerical calculations performed in Ref. [6] reveal that the χ particle production occurs for $g \gtrsim 0.06$ [28]. This condition is almost the same as in the oscillating inflation model in spite of the difficulty in producing the *massless* χ particle without the larger g compared with the $V(\phi, \chi) = m^2 \phi^2/2 + g^2 \phi^2 \chi^2/2$ model. The reason is simple. In Eq. (3.12), the massive χ particle production occurs when the initial value of the resonance term $g^2 \phi_0^2/m_\phi^2$ is much greater than unity $[(g^2 \phi_0^2/m_\phi^2)_i \gg 1]$ and \bar{m}_χ^2 is much smaller than this term $[\bar{m}_\chi^2 \ll (g^2 \phi_0^2/m_\phi^2)_i]$. In the case of $\phi_c = 10^{-3} M_{\text{pl}}$, $q = 0.1$, and $g = 0.05$, since $M = 3.96 \times 10^{-4} M_{\text{pl}}$ by Eq. (2.28), we obtain $(g^2 \phi_0^2/m_\phi^2)_i \approx 4 \times 10^3$. Although $\tilde{\phi}_0$ decreases by the cosmic expansion and m_ϕ becomes large in the vicinity of $\phi_0 = 0$, the massive χ particle production is possible as long as $g^2 \tilde{\phi}_0^2/m_\phi^2 \gg 1$ and $\bar{m}_\chi^2 \ll g^2 \tilde{\phi}_0^2/m_\phi^2$. Since the mass m_ϕ in this case is estimated as

$m_\phi \approx 10^{-5}M_{\text{pl}}$ for $\phi_0^2 \gg \phi_c^2$ and $m_\phi \approx 10^{-4}M_{\text{pl}}$ for $\phi_0^2 \ll \phi_c^2$ by Eq. (3.8), this is heavier than the mass $m \approx 10^{-6}M_{\text{pl}}$ in the $V(\phi, \chi) = m^2\phi^2/2 + g^2\phi^2\chi^2/2$ model. Even in the case where the normalized mass is $\bar{m}_\chi = 1$, this corresponds to the rather heavy particle whose mass is $m_\chi \gtrsim 10^{14}$ GeV. For the typical value $\phi_c \lesssim 10^{-3}M_{\text{pl}}$ in the oscillating inflation model, since m_ϕ is at least by one order larger than $m = 10^{-6}M_{\text{pl}}$, the massive χ particle whose mass is of order $m_\chi = 10^{14}$ GeV can be generated for the coupling g where the massless χ production occurs. As the coupling g increases, more massive particles are generated. In the case of $\phi_c = 10^{-3}M_{\text{pl}}$, we find that the production of the massive particle whose mass is of order $m_\chi = 10^{15}$ GeV relevantly occurs for the coupling $g \gtrsim 0.2$. Even the GUT scale bosons $m_\chi = 10^{16}$ GeV are produced in the initial stage of the oscillating inflation for the case of $g = 1$, although the achieved amount is small (See Fig. 14).

With the decrease of ϕ_c , the larger values of g are required in order to produce the massive χ particle as is the same with the massless χ particle. However, in the parameter range of g where the massless χ particles are sufficiently produced, we can also expect the generation of the superheavy particles needed for the success of the GUT baryogenesis. For example, in the case of $\phi_c = 10^{-4}M_{\text{pl}}$, numerical calculations indicate that χ particles up to the mass $m_\chi \lesssim 10^{14}$ GeV and $m_\chi \lesssim 10^{15}$ GeV are created for the coupling $g \gtrsim 0.07$ and $g \gtrsim 0.3$ respectively. It will be interesting to investigate how these superheavy bosons produced in the oscillating inflation model would affect the scenario of the GUT baryogenesis.

IV. CONCLUDING REMARKS AND DISCUSSIONS

In this paper we have considered parametric amplification of a scalar field χ coupled to an inflaton field ϕ as well as the enhancement of the fluctuation of the ϕ field in the oscillating inflation model. Since the inflaton potential $V(\phi)$ is the non-convex type where $d^2V/d\phi^2$ takes negative values in the regions not too far from the minimum of the potential, inflation is realized even during the oscillation of the inflaton field. The oscillating inflation model is characterized by the critical value of ϕ_c which indicates the scale of the core part of the potential. Inflation takes place while the ϕ field moves in the regions of $|\phi| \gtrsim \phi_c$. With the decrease of ϕ_c , the total amount of inflation becomes larger, because the duration during which the oscillating inflation takes place becomes longer. However, it is rather difficult to obtain the sufficient number of e -foldings such as $N \gtrsim 60$ only by the stage of the oscillating inflation, and the main contribution to the number of e -foldings is achieved by the ordinary slow-roll inflation which precedes the oscillating inflation. We have estimated the energy scale of the oscillating inflation M^4 by fitting the primordial density perturbation observed by COBE. The oscillating inflation terminates when the amplitude of the ϕ_0 field decreases to of the order ϕ_c by the expansion of the universe.

In this model, the fluctuation of the ϕ field grows during the oscillating inflation due to the existence of the non-convex part of the potential. We examined the growth of the fluctuation for two typical cases $\phi_c = 10^{-3}M_{\text{pl}}$ and $\phi_c = 10^{-4}M_{\text{pl}}$ including the back reaction effect of created particles based on the Hartree approximation. We have found that the wave modes inside the Hubble horizon are effectively enhanced, while the growth of long-wave modes is small as was shown in Ref. [19]. Although the growth rate of the fluctuation $\langle \delta\phi^2 \rangle$ in the initial stage of the oscillating inflation becomes larger with the decrease of ϕ_c , the final fluctuation is significantly suppressed. This indicates that

the generation of the ϕ particle itself plays a crucial role for the termination of parametric resonance. As for the final variance of the ϕ particle, we find the relation $\langle \delta\phi^2 \rangle_f \approx 0.1\phi_c^2$ for the case of $\phi_c \lesssim 10^{-3}M_{\text{pl}}$.

With regard to the χ field coupled to inflaton, the enhancement of the fluctuation mainly occurs when the ϕ_0 field moves nonadiabatically around the minimum of the potential. Since the mass m_ϕ for $|\phi_0| \lesssim \phi_c$ is approximately written as $m_\phi \approx M^2/\phi_c$, it is larger than that of the massive inflaton of the chaotic inflation model for the value of $\phi_c \lesssim 10^{-3}M_{\text{pl}}$. As m_ϕ becomes larger, this restricts the effective χ particle production. Moreover, the amplitude of the homogeneous inflaton field decreases faster than in the model of the massive inflaton. This means that the χ particle production is not possible unless we select rather large values of the coupling constant g . This tendency becomes stronger with the decrease of ϕ_c . For example, in the case of $\phi_c = 10^{-3}M_{\text{pl}}$ and $q = 0.1$, the coupling $g \gtrsim 0.05$ is required to yield the sufficient χ particle production, the value of which is two orders of magnitude larger than in the model of $V(\phi, \chi) = m^2\phi^2/2 + g^2\phi^2\chi^2/2$. For $g \gtrsim 0.05$, the back reaction effect onto the ϕ_0 field due to the χ particle production is significant, and finally terminates parametric resonance. The final variance is suppressed by the back reaction effect with the increase of g , and the maximal variance for the $\phi_c = 10^{-3}M_{\text{pl}}$ case is $\langle \chi^2 \rangle_f \approx 10^{-5}M_{\text{pl}}^2$ when $g \approx 0.3$. In the case of $\phi_c = 10^{-4}M_{\text{pl}}$, we need $g \gtrsim 0.1$ for an effective resonance, although the χ particle production occurs for $g \gtrsim 0.01$. The maximal variance for the $\phi_c = 10^{-4}M_{\text{pl}}$ case is $\langle \chi^2 \rangle_f \approx 10^{-6}M_{\text{pl}}^2$ when $g \approx 0.5$. As ϕ_c decreases further, we no longer expect the growth of the χ fluctuation with the natural coupling $g \lesssim 1$. Moreover, since the final fluctuation of the ϕ particle is also suppressed, the small ϕ_c such as $\phi_c \lesssim 10^{-5}M_{\text{pl}}$ is not favorable for the production of a sufficient amount of ϕ and χ particles.

In the oscillating inflation model, the superheavy χ particle with mass greater than $m_\chi = 10^{14}$ GeV can be copiously produced. For example, in the case of $\phi_c = 10^{-3}M_{\text{pl}}$, we find that the χ particle whose mass is of order 10^{14}GeV and 10^{15}GeV is effectively enhanced for $g \gtrsim 0.05$ and $g \gtrsim 0.2$ respectively. The GUT scale boson $m_\chi \sim 10^{16}$ GeV can be also generated in the initial stage if the coupling g is of order unity, although the final amount is small. In Ref. [22], the authors considered the GUT scale baryogenesis with the massive inflaton assuming that some of the initial inflaton energy is efficiently transferred to the bosons with the mass 10^{14} GeV. These massive bosons decay into lighter particles, and produce the net baryon number. Although we do not consider such a decaying process in this paper, it is of interest how the production of the superheavy particle would affect the baryon asymmetry in the universe.

In this paper we make use of the Hartree approximation, which is essentially the mean field approximation. This does not include the rescattering effect which becomes important as χ particles are sufficiently produced. The scattering between ϕ and χ particles may reduce the final amount of fluctuations. For a complete study including the nonlinear effect of the particle production, we should perform the lattice simulations and compare them with the mean field approximation performed in this paper.

Finally, we comment on the case where the metric perturbation is included. In the single field case, even if we consider the effect of the metric perturbation, the $V^{(2)}(\phi_0)$ term in Eq. (2.9) is much more important than the $2\kappa^2(V/H)$ term which is the gravitational origin. Hence the situation is almost the same as the case when the metric perturbation is neglected. As we have showed, the long-wave modes are not significantly enhanced compared with the modes inside the Hubble Horizon in the single-field case. This result is consistent with other inflation models

in the single-field case [29]. In the two-field case, it was suggested that super-Hubble metric perturbations can be amplified in broad classes of models [30]. In the present model, since the χ fluctuation of super-Hubble modes will be exponentially suppressed for large values of $q \gg 1$ [31], we may not expect the enhancement of super-Hubble metric perturbations significantly. However, it is worth investigating to investigate these issues including mode-mode coupling between field and metric perturbations, because this may lead to some imprints on the spectrum of density perturbations. These issues are under consideration.

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- [1] A. H. Guth, Phys. Rev. D **23**, 347 (1981); K. Sato, Mon. Not. R. Astron. Soc. **195**, 467 (1981); Phys. Lett. **99B**, 66 (1981).
 - [2] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Redwood City, California, 1990); A. D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood, Chur, Switzerland, 1990).
 - [3] A. D. Dolgov and A. D. Linde, Phys. Lett. **116B**, 329 (1982); L. F. Abbott, E. Fahri, and M. Wise, Phys. Lett. **117B**, 29 (1982).
 - [4] J. Traschen and R. H. Brandenberger, Phys. Rev. D **42**, 2491 (1990); Y. Shatanov, J. Traschen, and R. H. Brandenberger, Phys. Rev. D **51**, 5438 (1995).
 - [5] L. Kofman, A. Linde, and A. A. Starobinsky, Phys. Rev. Lett. **73**, 3195 (1994).
 - [6] S. Khlebnikov and I. I. Tkachev, Phys. Lett. **B390**, 80 (1997); S. Khlebnikov and I. I. Tkachev, Phys. Rev. Lett. **79**, 1607 (1997).
 - [7] T. Prokopec and T. G. Roos, Phys. Rev. D **55**, 3768 (1997); B. R. Greene, T. Prokopec, and T. G. Roos, Phys. Rev. D **58**, 6484 (1997).
 - [8] L. Kofman, A. Linde, and A. A. Starobinsky, Phys. Rev. D **73**, 3258 (1997).
 - [9] D. Boyanovsky, H. J. de Vega, R. Holman, D. S. Lee, and A. Singh, Phys. Rev. D **51**, 4419 (1995); D. Boyanovsky, M. D'Attanasio, H. J. de Vega, R. Holman, D. S. Lee, and A. Singh, Phys. Rev. D **52**, 6805 (1995); D. Boyanovsky, H. J. de Vega, R. Holman, D. S. Lee, and A. Singh, J. F. J. Salgado, Phys. Rev. D **54**, 7570 (1996); D. Boyanovsky, D. Cormier, H. J. de Vega, R. Holman, A. Singh, and M. Srednicki, Phys. Rev. D **56**, 1939 (1997); D. Boyanovsky, R. Holman, S. Prem Kumar, Phys. Rev. D **56**, 1958 (1997).
 - [10] D. I. Kaiser, Phys. Rev. D **53**, 1776 (1995); *ibid* D **56**, 706 (1997).
 - [11] S. Khlebnikov and I. I. Tkachev, Phys. Rev. Lett. **77**, 219 (1996).
 - [12] D. T. Son, Phys. Rev. D **54**, 3745 (1996).
 - [13] J. Baacke, K. Heitmann, C. Pätzold, Phys. Rev. D **55**, 2320 (1997); *ibid* D **56**, 6556 (1997); J. Baacke and C. Pätzold, hep-ph/9906417.
 - [14] P. B. Greene, L. Kofman, A. Linde, and A. A. Starobinsky, Phys. Rev. D **56**, 6175 (1997).
 - [15] J. Garcia-Bellido and A. Linde, Phys. Rev. D **57**, 6075 (1998).
 - [16] S. Tsujikawa, K. Maeda, and T. Torii, Phys. Rev. D **60**, 123505 (1999); see also B. A. Bassett and S. Liberati, Phys. Rev. D **58**, 021302 (1998); S. Tsujikawa, K. Maeda, and T. Torii, Phys. Rev. D **60**, 063515 (1999); S. Tsujikawa, K. Maeda, and T. Torii, hep-ph/9910214, to appear in Phys. Rev. D.
 - [17] T. Damour and V. F. Mukhanov, Phys. Rev. Lett. **80**, 3440 (1998).
 - [18] A. R. Liddle and A. Mazumdar, Phys. Rev. D **58**, 083508 (1998).
 - [19] A. Taruya, Phys. Rev. D **59**, 103505 (1999).
 - [20] L. Kofman, A. Linde, and A. A. Starobinsky, Phys. Rev. Lett. **76**, 1011 (1996); I. I. Tkachev, Phys. Lett. **B376**, 35 (1996); S. Kasuya and M. Kawasaki, Phys. Rev. D **56**, 7597 (1997).

- [21] S. Khlebnikov, L. Kofman, A. Linde, and I. I. Tkachev, Phys. Rev. Lett. **81**, 2012 (1998); I. I. Tkachev, S. Khlebnikov, L. Kofman, A. Linde, Phys. Lett. **B440**, 262 (1998); S. Kasuya and M. Kawasaki, Phys. Rev. D **58**, 083516 (1998).
- [22] E. W. Kolb, A. D. Linde, and A. Riotto, Phys. Rev. Lett. **77**, 3716 (1996); E. W. Kolb, A. Riotto, and I. I. Tkachev, Phys. Lett. **B423**, 348 (1998).
- [23] In this paper we do not consider the nonminimal coupling of the ϕ or χ field with the scalar curvature R . However, there is a possibility that the nonminimal coupling would lead to the gravitational particle production as in Ref. [16].
- [24] D. Cormier and R. Holman, Phys. Rev. D **60**, 041301 (1999).
- [25] D. H. Lyth, Phys. Rev. D **31**, 1792 (1985); A. R. Liddle and D. H. Lyth, Phys. Rep. **231**, 1 (1993).
- [26] N. W. Mac Lachlan, *Theory and Applications of Mathieu Functions* (Dover, New York, 1961).
- [27] S. Tsujikawa, in preparation.
- [28] This criterion was also obtained by analytic estimates performed in Ref. [8].
- [29] H. Kodama and T. Hamazaki, Prog. Theor. Phys. **96**, 949 (1996); Y. Nambu and A. Taruya, Prog. Theor. Phys. **97**, 83 (1997); F. Finelli and R. Brandenberger, Phys. Rev. Lett. **82** 1362 (1999); M. Parry and R. Easter, Phys. Rev. D **59** 061301 (1999); S. Tsujikawa and B. A. Bassett, hep-ph/0003068.
- [30] B. A. Bassett, D. I. Kaiser, and R. Maartens, Phys. Lett. **B455**, 84 (1999); B. A. Bassett, F. Tamburini, D. I. Kaiser, and R. Maartens, Nucl. Phys. **B561**, 188 (1999); B. A. Bassett and F. Viniegra, hep-ph/9909353; B. A. Bassett, C. Gordon, R. Maartens, and D. I. Kaiser, Phys. Rev. D. 061302(R) (2000); F. Finelli and R. Brandenberger, hep-ph/0003172.
- [31] P. Ivanov, Phys. Rev. D **61**, 023505 (2000); K. Jedamzik and G. Sigl, Phys. Rev. D **61**, 023519 (2000).

Figure Captions

- FIG. 1: The potential for $q = 0.1$ in the oscillating inflation model, which is composed of the non-convex region $|\phi| \gtrsim \phi_c$ and the core region $|\phi| \lesssim \phi_c$.
- FIG. 2: The evolution of the ϕ_0 field in the case of $\phi_c = 10^{-3}M_{\text{pl}}$, $q = 0.1$, and $g = 0$. The stage of the oscillating inflation starts from $\phi_0 \approx 1.4 \times 10^{-2}M_{\text{pl}}$, and ends at $\bar{t} \approx 0.37$ when the amplitude of the ϕ_0 field becomes of order ϕ_c .
- FIG. 3: The evolution of the real part of the fluctuation $\delta\phi_k$ for two momentum modes of $\bar{k} = 0.1$ and $\bar{k} = 100$ in the case of $\phi_c = 10^{-3}M_{\text{pl}}$, $q = 0.1$, and $g = 0$. For the long wave mode $\bar{k} = 0.1$, the growth of the fluctuation is weak, but for the mode $\bar{k} = 100$, the fluctuation grows rapidly until the back reaction effect becomes significant at $\bar{t} \approx 0.12$.
- FIG. 4: The evolution of $\langle\delta\phi^2\rangle$ in the case of $\phi_c = 10^{-3}M_{\text{pl}}$, $q = 0.1$, and $g = 0$. The fluctuation reaches the maximum value $\langle\delta\phi^2\rangle_f \approx 10^{-7}M_{\text{pl}}^2$ at $\bar{t} \approx 0.12$.
- FIG. 5: The evolution of $\langle\delta\phi^2\rangle$ in the case of $\phi_c = 10^{-4}M_{\text{pl}}$, $q = 0.1$, and $g = 0$. Although the initial growth rate is larger than in the case of $\phi_c = 10^{-3}M_{\text{pl}}$, the fluctuation soon reaches the maximum value $\langle\delta\phi^2\rangle_f \approx 8.0 \times 10^{-10}M_{\text{pl}}^2$, which is by two orders smaller than in the case of $\phi_c = 10^{-3}M_{\text{pl}}$.
- FIG. 6: The evolution of the fluctuation $\langle\chi^2\rangle$ in the case of $\phi_c = 10^{-3}M_{\text{pl}}$ and $q = 0.1$ for $g = 0.03$ (bottom) and $g = 0.07$ (top). For $g = 0.03$, the maximum value of the fluctuation is $\langle\chi^2\rangle_f = 2.3 \times 10^{-11}M_{\text{pl}}^2$; and for $g = 0.07$, $\langle\chi^2\rangle_f = 2.4 \times 10^{-7}M_{\text{pl}}^2$.
- FIG. 7: The evolution of the ϕ_0 field in the case of $\phi_c = 10^{-3}M_{\text{pl}}$, $q = 0.1$, and $g = 0.07$. The coherent oscillation is a bit broken at $\bar{t} \approx 0.12$ due to the increase of $\langle\delta\phi^2\rangle$. With the increase of $\langle\chi^2\rangle$, this also changes the frequency of the ϕ_0 field at $\bar{t} \approx 0.28$.
- FIG. 8: The evolution of $\langle\delta\phi^2\rangle$ in the case of $\phi_c = 10^{-3}M_{\text{pl}}$, $q = 0.1$, and $g = 0.07$. With the growth of $\langle\chi^2\rangle$, this assists the ϕ particle production, and the final variance $\langle\delta\phi^2\rangle_f = 2.7 \times 10^{-6}M_{\text{pl}}^2$ becomes larger than in the case of $\phi_c = 10^{-3}M_{\text{pl}}$ and $g = 0$ by one order of magnitude.
- FIG. 9: The final variance $\langle\chi^2\rangle_f$ as the function of g in the case of $\phi_c = 10^{-3}M_{\text{pl}}$ and $q = 0.1$. We find that $\langle\chi^2\rangle_f$ takes the maximum value $\langle\chi^2\rangle_{\text{max}} \approx 10^{-5}M_{\text{pl}}^2$ when $g \approx 0.3$.

- FIG. 10: The evolution of the real part of the fluctuation χ_k for two momentum modes of $\bar{k} = 0$ and $\bar{k} = 100$ in the case of $\phi_c = 10^{-3}M_{\text{pl}}$, $q = 0.1$, and $g = 0.1$. The long wave mode $\bar{k} = 0$ is strongly enhanced compared with the wave mode $\bar{k} = 100$.
- FIG. 11: The evolution of the fluctuation $\langle\chi^2\rangle$ in the case of $\phi_c = 10^{-4}M_{\text{pl}}$, $q = 0.1$, and for $g = 0.07$ (bottom), $g = 0.1$ (middle), and $g = 0.5$ (top) respectively. The final variances are $\langle\chi^2\rangle_f = 1.2 \times 10^{-10}M_{\text{pl}}^2$, $2.5 \times 10^{-9}M_{\text{pl}}^2$, and $1.3 \times 10^{-6}M_{\text{pl}}^2$ respectively.
- FIG. 12: The evolution of the ϕ_0 field in the case of $\phi_c = 10^{-4}M_{\text{pl}}$, $q = 0.1$, and $g = 0.5$. While the back reaction effect of ϕ particles can be negligible, the increase of $\langle\chi^2\rangle$ strongly affects the evolution of the ϕ_0 field from $\bar{t} \approx 0.17$.
- FIG. 13: The evolution of $\langle\chi^2\rangle$ for $\phi_c = 10^{-3}M_{\text{pl}}$, $q = 0.1$, and $g = 0.05$ in two cases of $m_\chi = 3 \times 10^{14}$ GeV (top) and $m_\chi = 1 \times 10^{15}$ GeV (bottom). The χ particle whose mass is of order $m_\chi = 10^{14}$ GeV is generated, but the production of the massive particle with $m_\chi \gtrsim 10^{15}$ GeV is hardly expected for $g = 0.05$.
- FIG. 14: The evolution of $\langle\chi^2\rangle$ for $\phi_c = 10^{-3}M_{\text{pl}}$ and $q = 0.1$, in two cases of $g = 0.5$, $m_\chi = 1 \times 10^{15}$ GeV (top); and $g = 1$, $m_\chi = 1 \times 10^{16}$ GeV (bottom). In the case of $g = 0.5$, the χ particle whose mass is of order $m_\chi = 10^{15}$ GeV is effectively enhanced. In the case of $g = 1$, the GUT scale boson $m_\chi = 10^{16}$ GeV is a little produced in the initial stage of the oscillating inflation.

Fig. 1

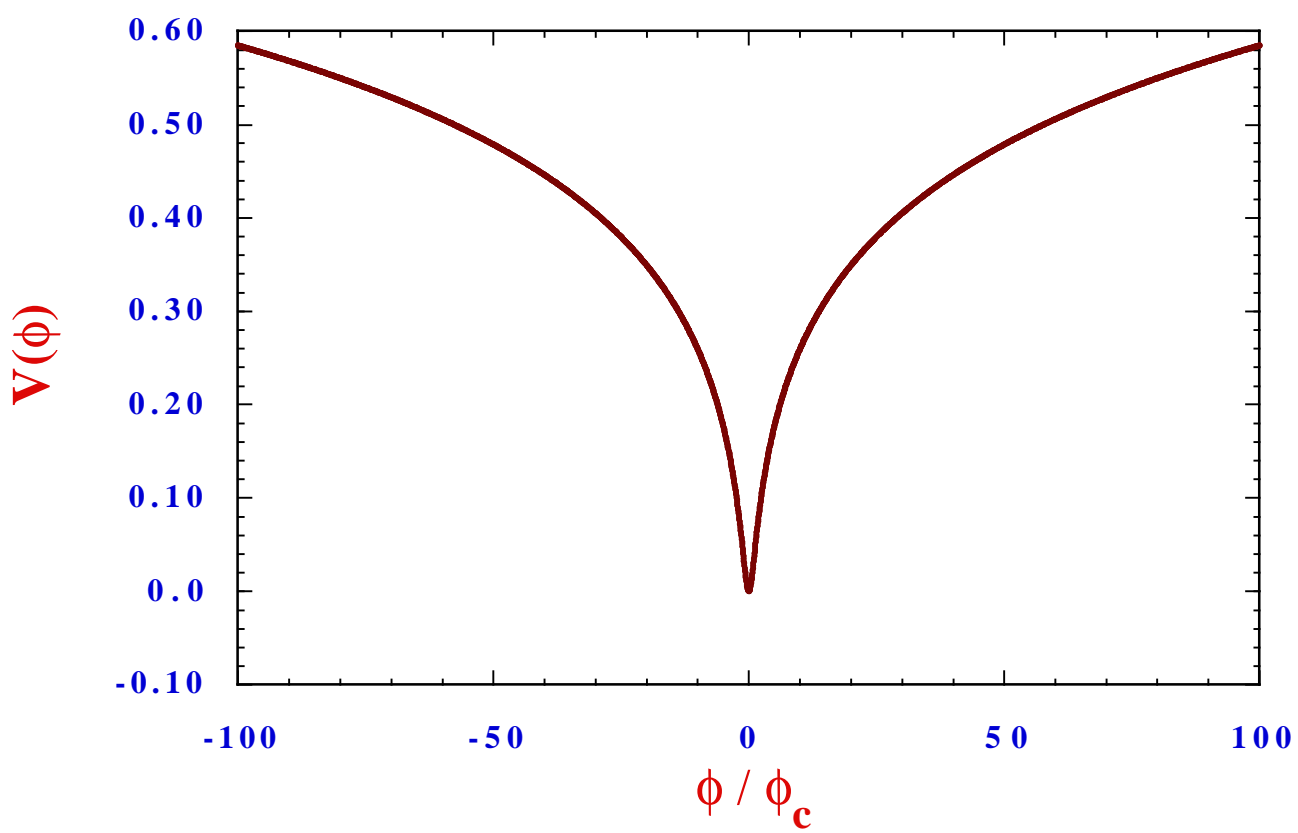


Fig. 2

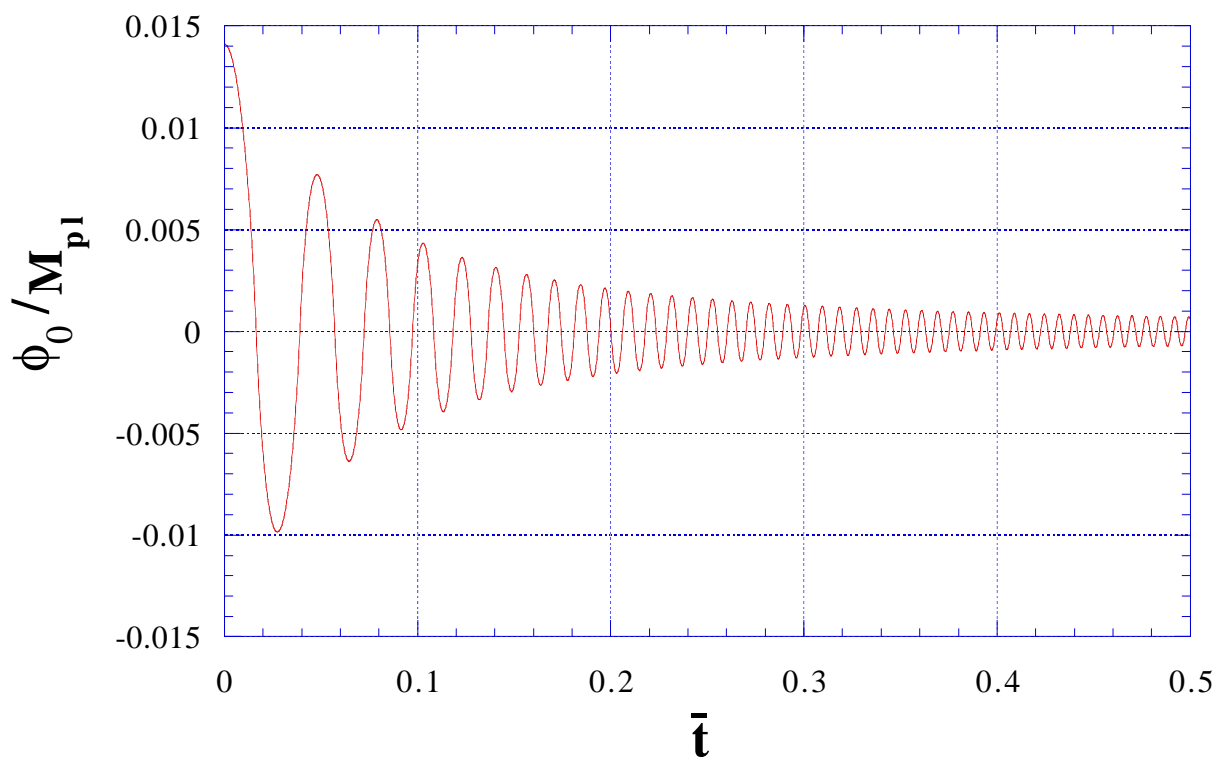


Fig. 3

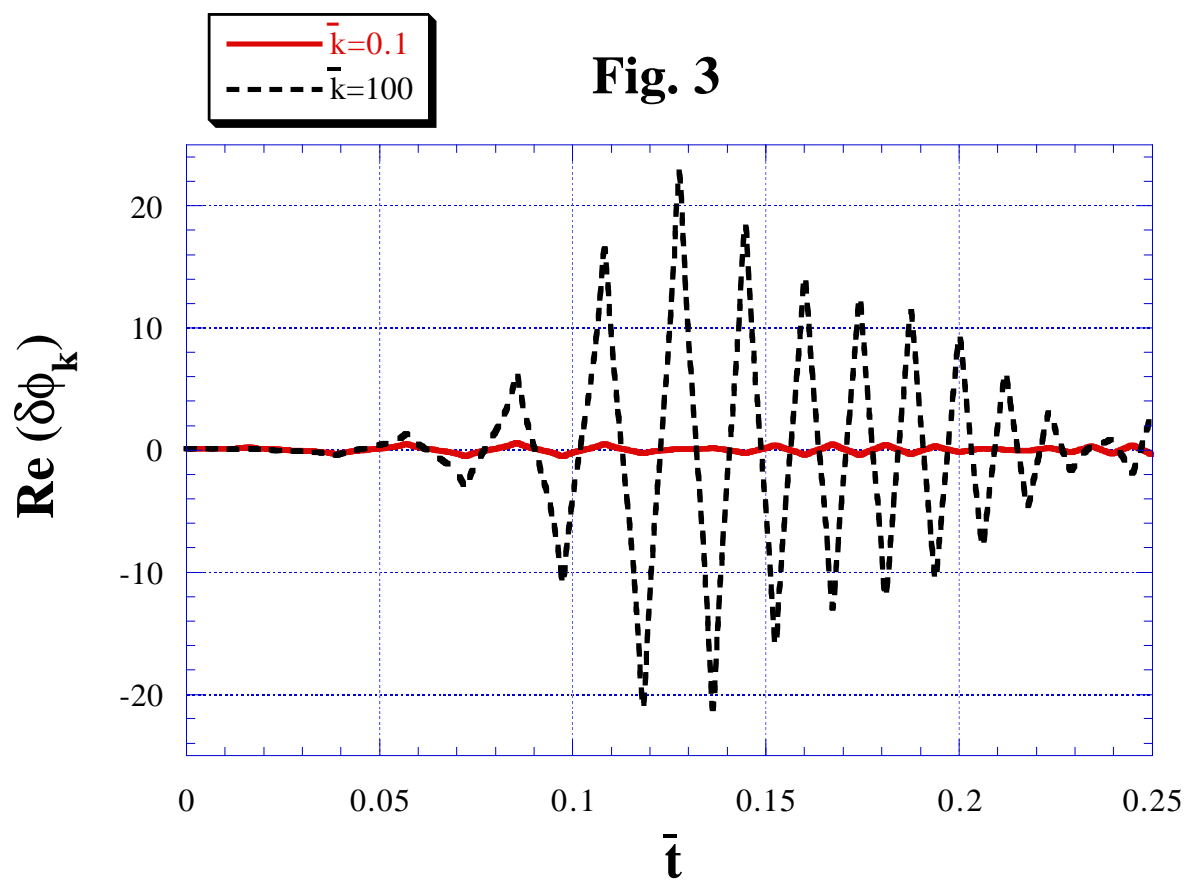


Fig. 4

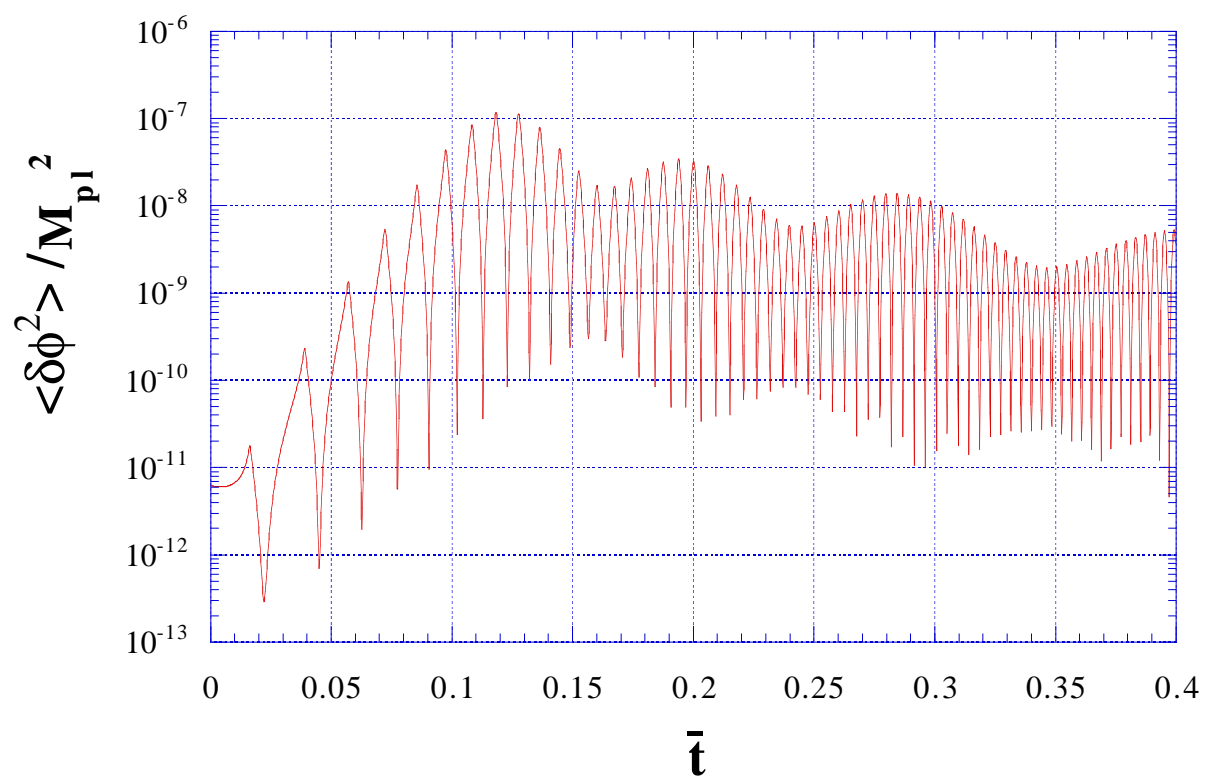


Fig. 5

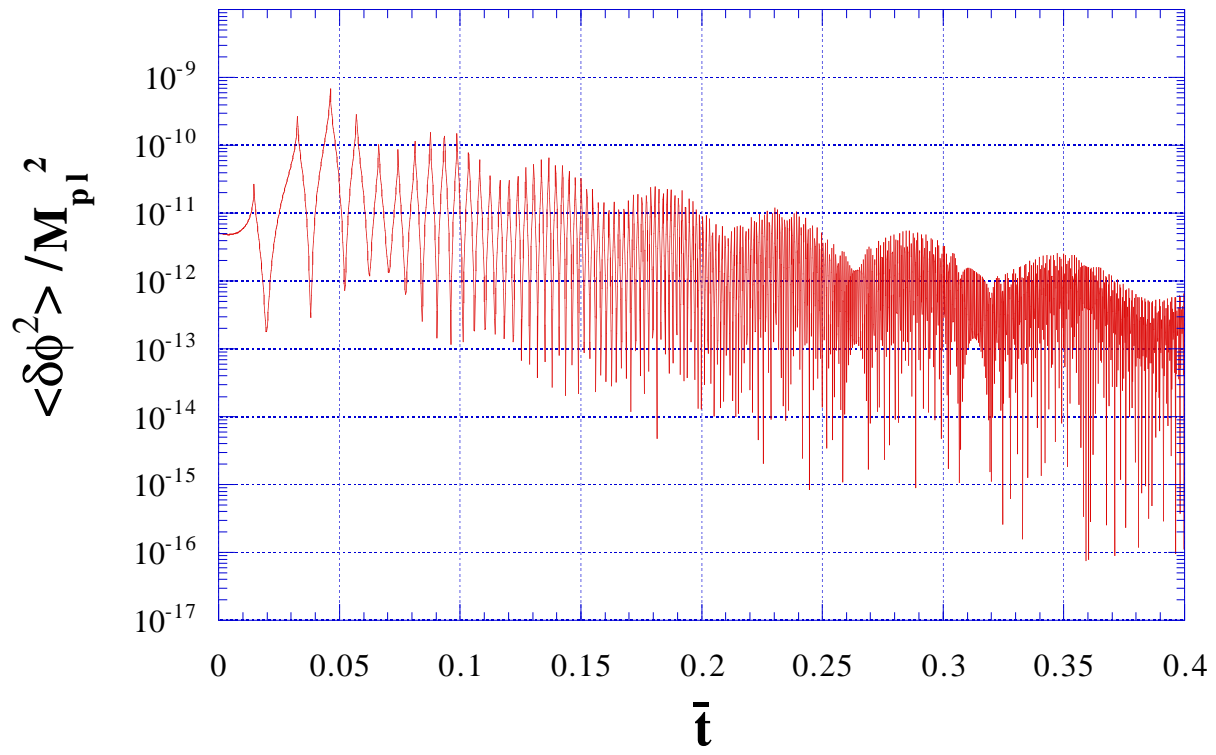


Fig. 6

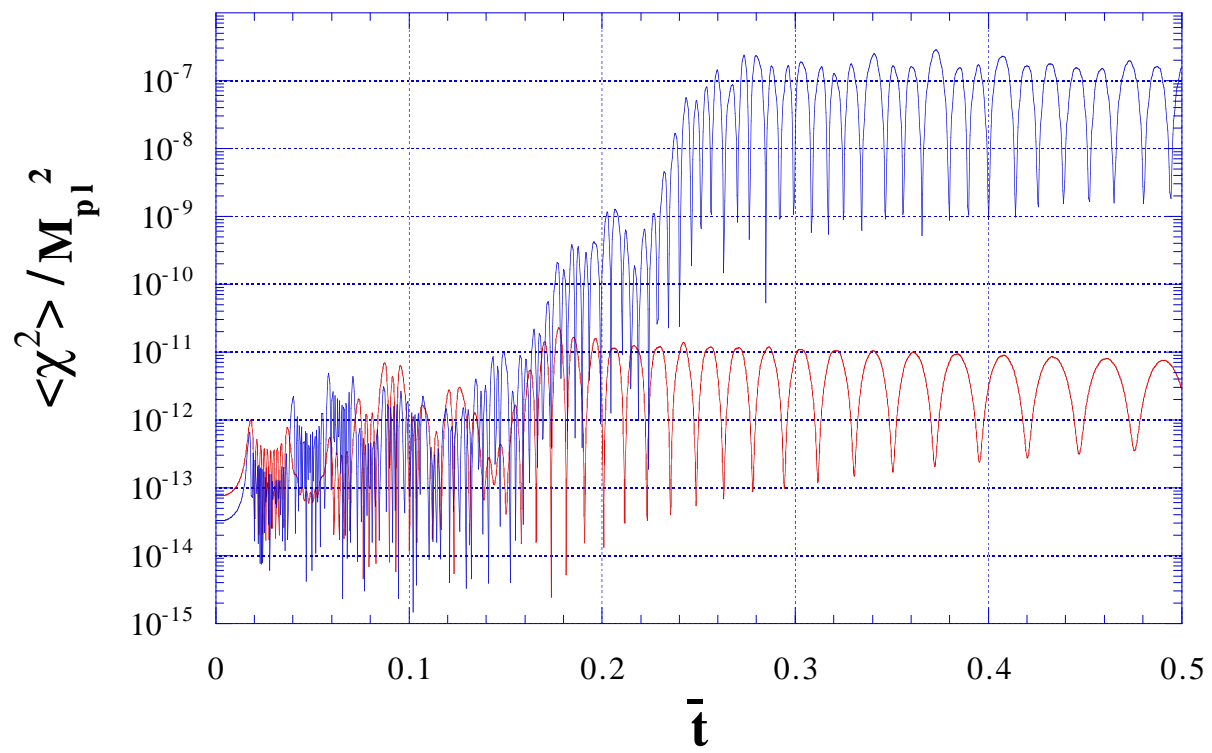


Fig. 7

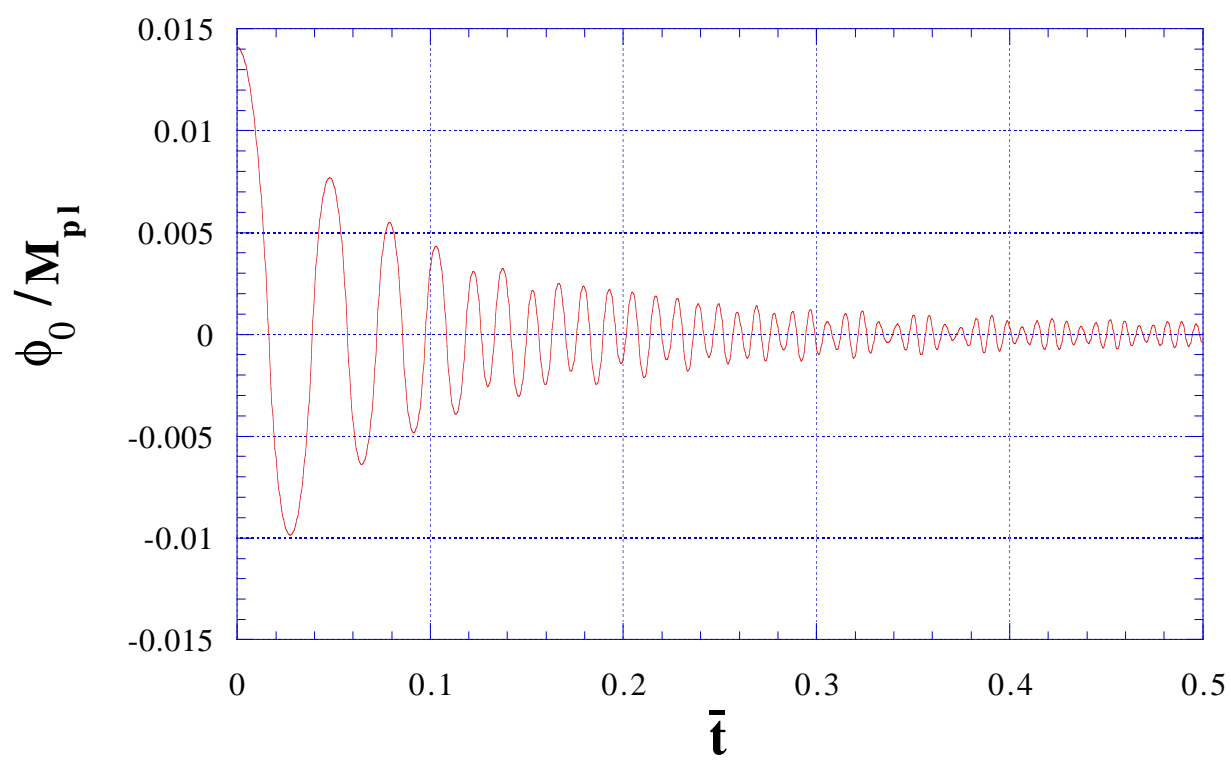


Fig. 8

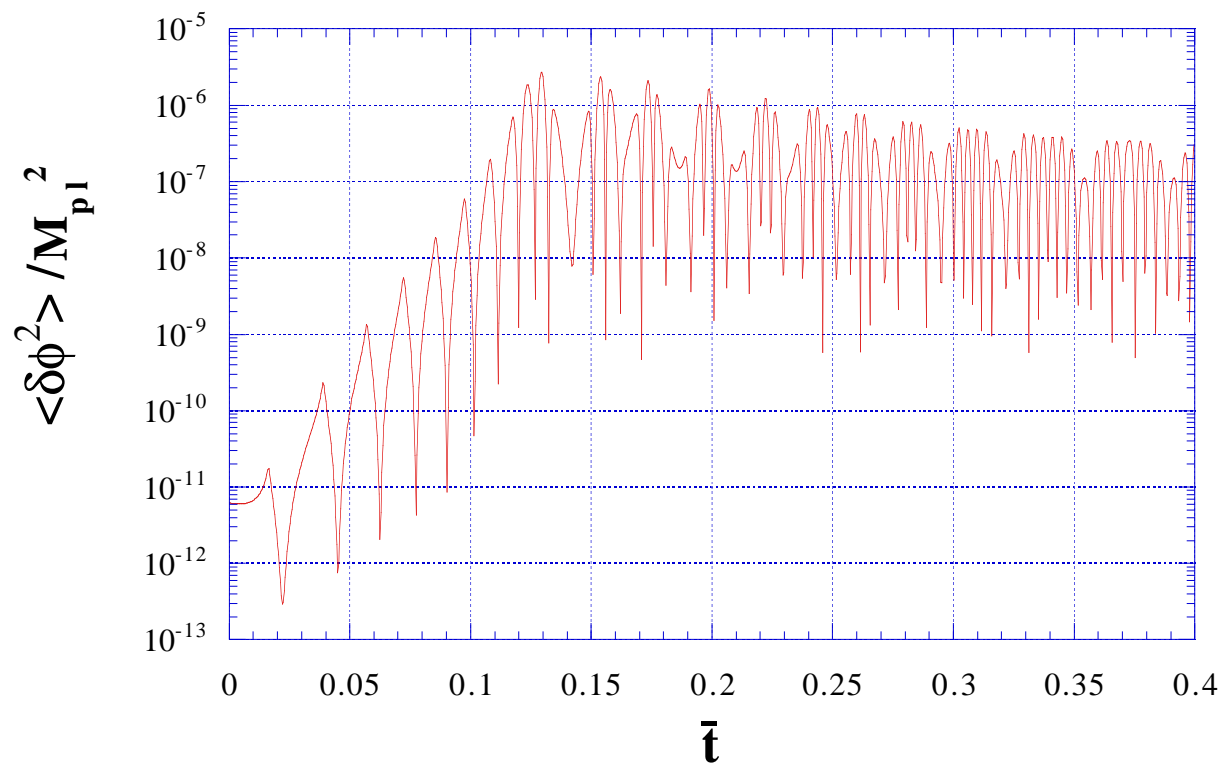
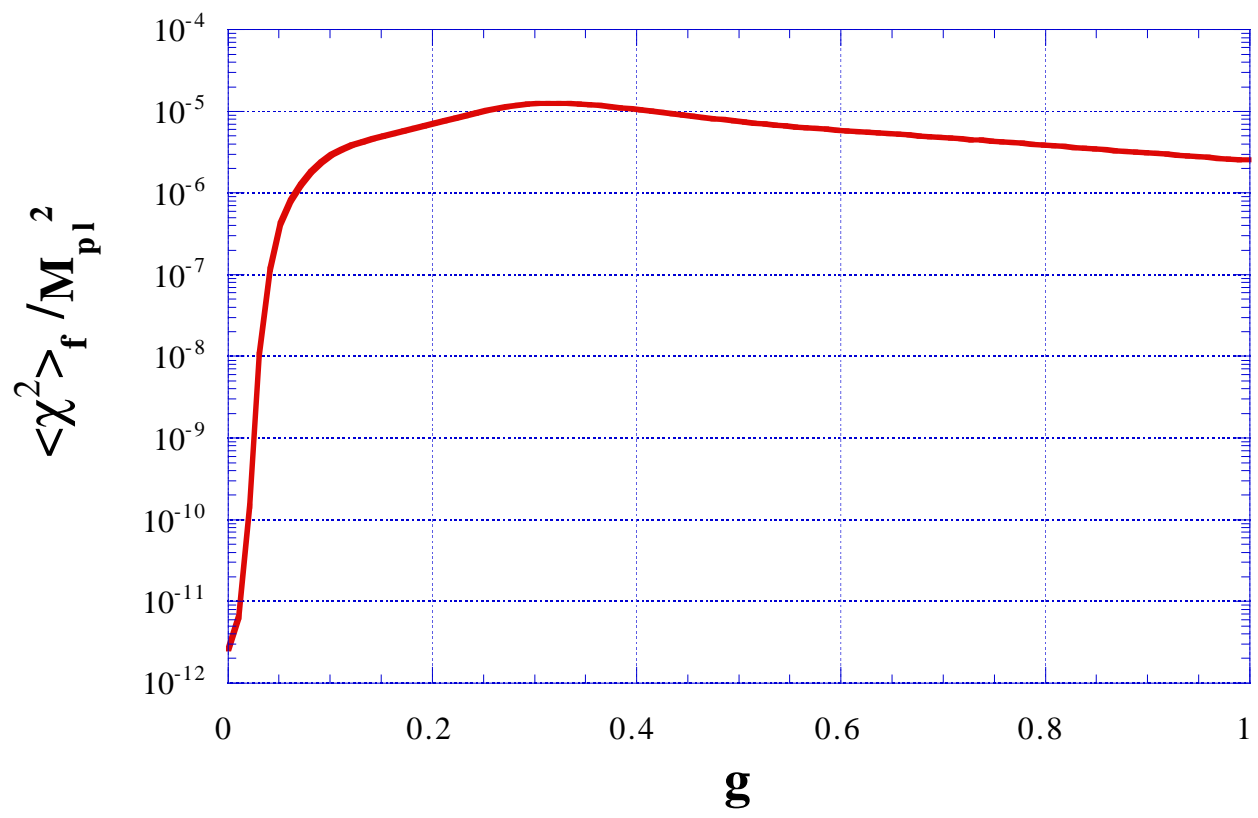


Fig. 9



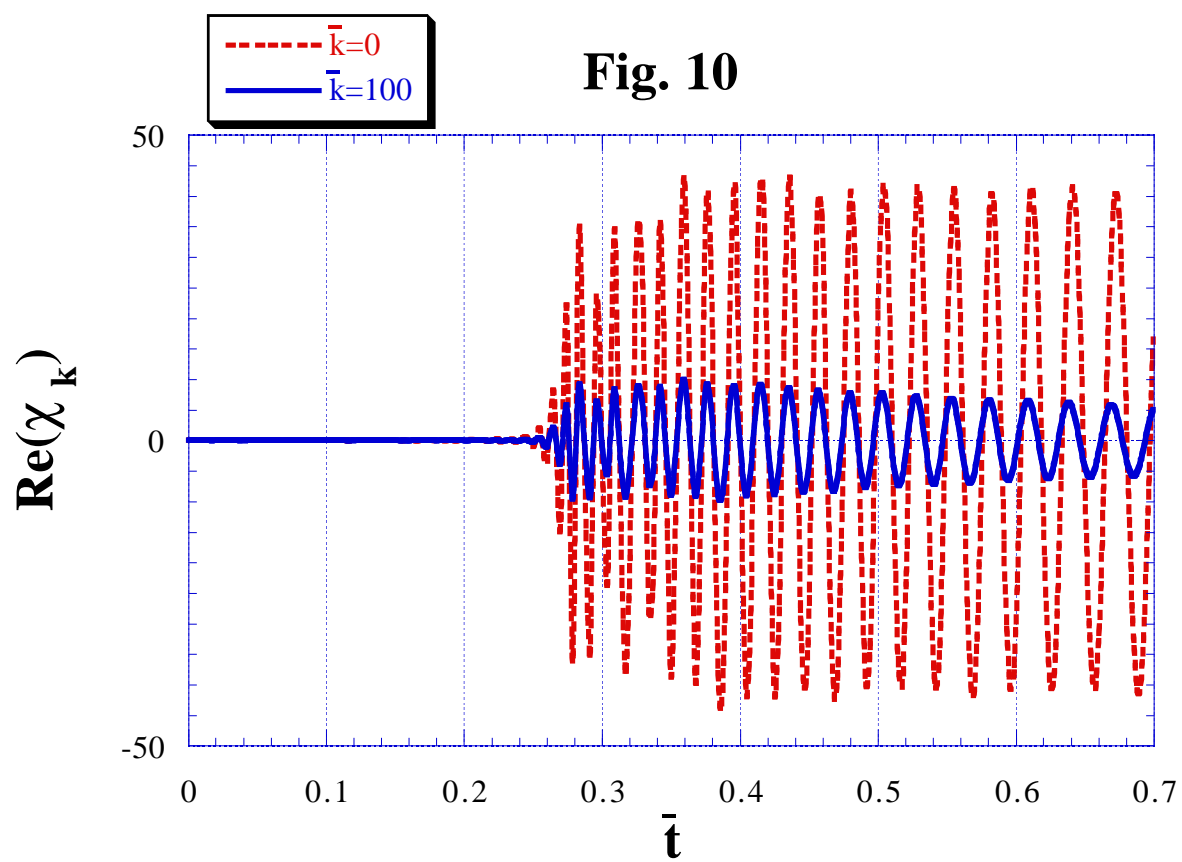


Fig. 11

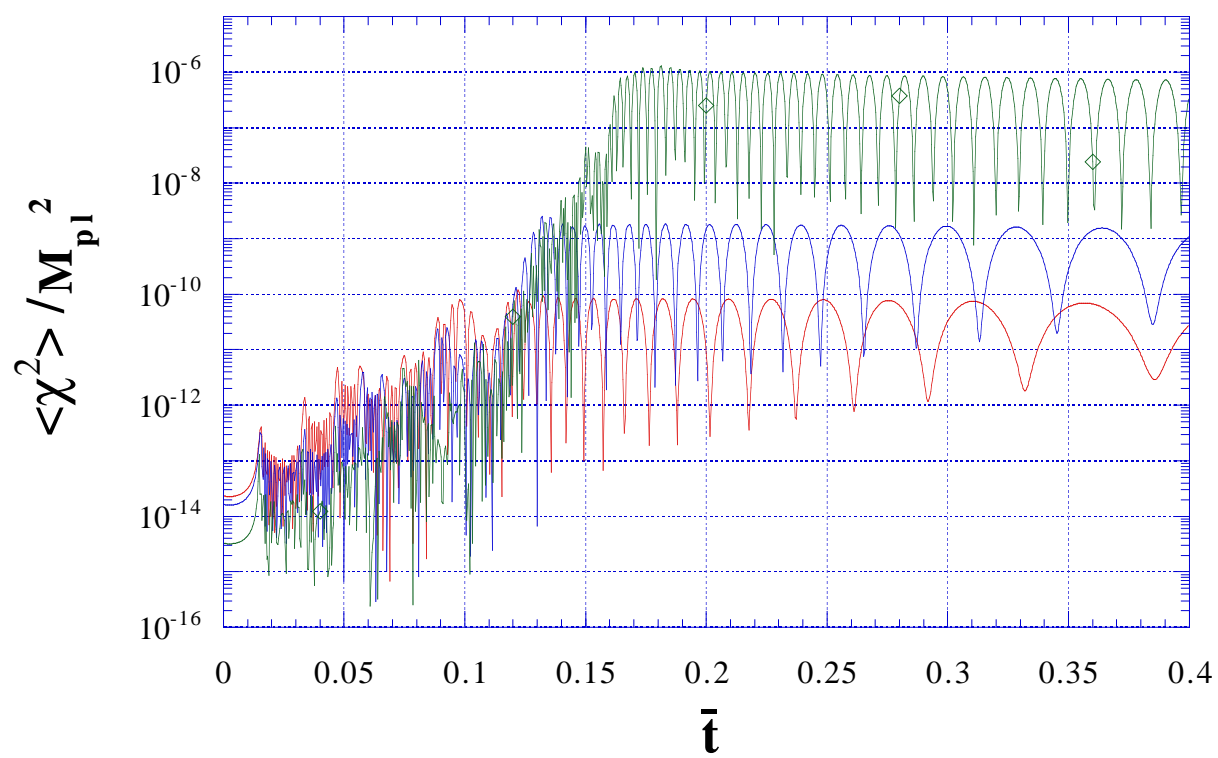


Fig. 12

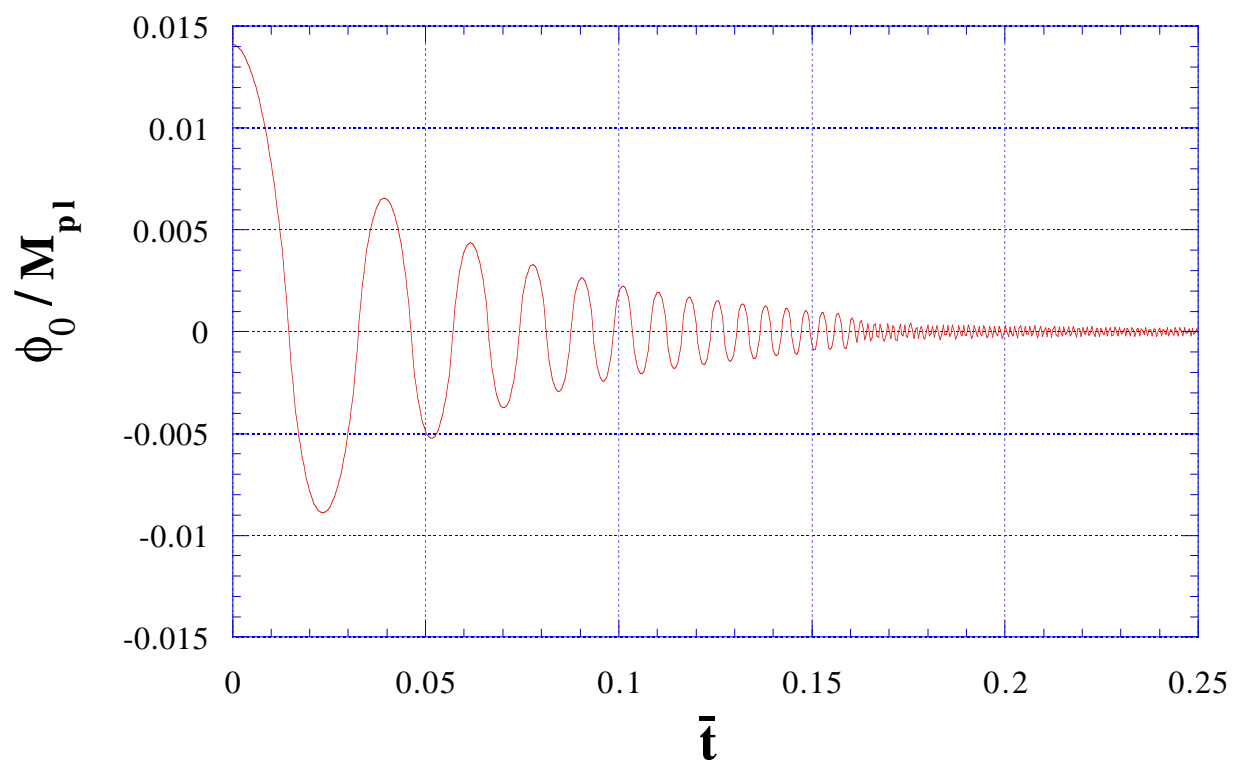


Fig. 13

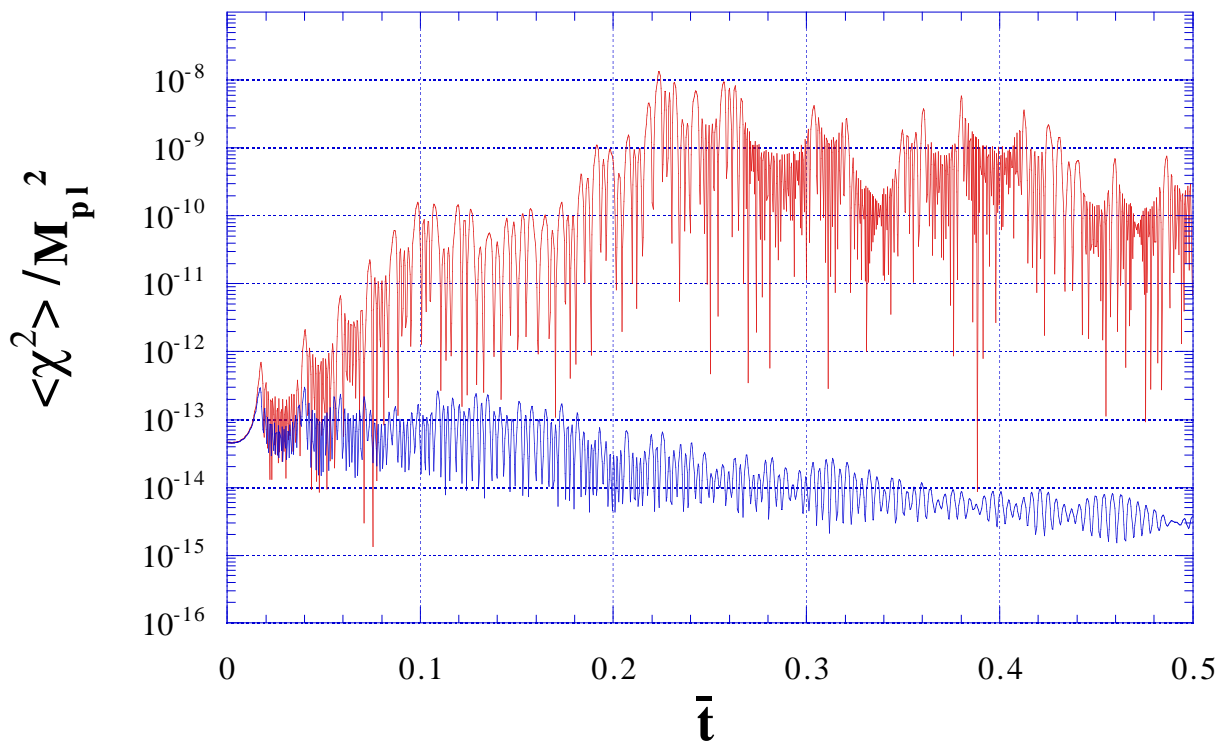


Fig. 14

